Philosophical Prolegomena of Geometry

and solution of its postulates

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Preface

The work that we are publishing today has a double aim: to show the relationship, in philosophical terms, of geometry—and mathematics in general—with the other sciences; then to found geometry on a truly scientific basis, to arrive at a complete solution of its postulates. We will first say a few words about this last, purely mathematical part. We will quickly sketch the history of our ideas. Our aim, in doing so, is not to follow a vain satisfaction of self-love. We believed that by indicating the manner in which we came to discover the principles which we expound, we would give them a new degree of evidence. [VI]

"There is no doubt," said Legendre, "that it is due to the imperfection of vulgar language and the difficulty of giving a good definition of the straight line, that geometers have had little success when they wanted to deduce this theorem (that the sum of the angles of a triangle is equal to two rights) solely from the notion of the equality of triangles contained in the first book of the elements."

If it is true that the conjectures of a man of genius are like a presentiment of the truth, it was necessary to perfect the mathematical language, by rejecting any term that had not been defined, to then proceed to the search for a good definition of the straight line; this definition had to be such that we could deduce, by demonstrative means, the proposition that *the straight line is the shortest path between two points*, as well as the postulates which complete it: *between two points we can only draw a straight line; when two portions of lines coincide, the lines themselves coincide throughout their entire extent.*

A first and important step was taken, if one could establish the absurdity of a *determinate* number of shortest paths between two points. Since, if [VII] in between two points A and B there exists a *determinate* number of shortest lines, there must exist the *same* number in between points C and B regardless of which of the two; which contradicts the hypothesis.[1] As for the principle which justifies this reasoning, it dominates all of geometry and of the other sciences; it is the principle of *homogeneity*, that is to say, this property of scientific space of being able to be indefinitely and indifferently divided into similar parts. To discover this principle meant at the same time to discover, that the straight line is a *homogeneous line*, that the plane is a *homogeneous surface*.

By starting from this notion of space, the plane and the straight, and of the corollary notions of size and shape, it was quite easy to deduce the elementary theorems of the straight line, the plane and of similitude; but there remained some difficulties surrounding the theory of parallelism, and to find a good definition of parallels.

Nonetheless, induction could facilitate the solution of this new problem. The new definitions of the line and the plane once acquired, [VIII] by investigating how they prevailed over the ordinary definitions, we should be on the path to the rules of *geometric definition*. This entirely critical work highlighted the *genetic* character of the new definitions, and the absence of this character in the old definitions, as well as in that of the old definitions of parallels. The problem therefore included a more precise statement. What is the general means of generating parallels?

The role that parallelism plays in the theory of similitude, drove me naturally to the idea of generating parallels in the same manner of generation as similar figures, that-is-to-say figures *which have the same shape but differ in size*. Now, the generative process offered itself here, so to speak: it was the magnification or diminution of space. In fact, if from any point we begin to enlarge or shrink space, any straight line it contains moves parallel to

[1] See the note on page "77", 3rd example.

itself. The parallels could therefore be defined as: *straight lines similarly placed in space*. But, space being homogeneous, any two lines occupy similar positions. It was therefore necessary to take a fixed position as a point of comparison. [IX] The variable position of the line around one of its points defined its *direction*, and the theory of parallels was definitively established.

It is on these principles that we have based the demonstration, so far sought in vain, of the postulates. These principles have also made it possible to considerably simplify the theory of similarity in general.

Could this work be of educational interest? In our thinking, yes. If the principles that we propose, and consequently the demonstrations for which they serve as a basis, were more obscure than the things to be demonstrated, we would have missed our objective. Any impractical solution to an elementary difficulty is not one in our eyes.[2]

For several years already we have been in possession of the purely mathematical ideas that we have just presented.[3] [X] The philosophical issues that can be raised in connection with the method and certainty of science preoccupied us at the same time; and, in our mind, we had united our research on the principles of geometry, and those on the fundamental principles of the human sciences. However, it was by no means our intention if we gave the solution of the postulates, to extend ourselves as much as we do today, on the philosophical questions. But later, upon reading a short dissertation by the learned Doctor Ueberweg, a dissertation conceived from this double point of view, we believed it was necessary to also follow this path. From this it comes that the philosophical part, general and special, holds a considerable place in our work, while the part which deals with the postulates, serves, so to speak, only to confirm the former.

The fundamental idea is quite a lot more simple.

Among the science said to be positive, we commonly distinguish the inductive sciences and the deductive sciences. Among the latter, mathematics is certainly included. However, on the one hand, there is controversy over whether or not mechanics is part of [XI] these sciences; and on the other, we believe we must reserve a separate place for arithmetic and algebra as they do not need, it is claimed, axioms or postulates. It is these distinc-

[2]This is not to say, however, that in teaching we should not, in certain cases, postpone the exposition of the principles of a science, and reserve it for the moment when the student is already more or less familiar with the subject it deals with. That's a completely different question. Thus, in order not to stray from the subject that occupies us, we could leave to the end of the geometry course the demonstration of the principles of similarity, isogeneity and homogeneity, but from the start use their consequences, that is to say, apply them.

[3]We had even written a few articles on this subject in the Annals of Public Education, published in Verviers, year 1857.

tions that we aim to destroy. We have sought to show that the positive sciences all proceed uniformly. *Observation* provides them with data; by *induction*, we rise to a *hypothetical principle* from which *scientific experience* draws consequences;[4] it is experience which properly serves to build science. The certainty of science arises from the logical legitimacy of its method—i.e. subjective certainty—and from the agreement of its results with observed facts—i.e. objective certainty. Science is true when its hypotheses are both subjective and objective.[5]

Every philosophical problem, however restricted it may seem, nevertheless has, in general, a very high scope, and is in intimate relationships with other more serious questions. [XII] We could ask ourselves here whether the moral and philosophical sciences should follow in step with the sciences said to be positive. There are some among them, and psychology is among this number, which unquestionably admit the same principles. But there are others, metaphysics, for example, where, it seems, they would no longer be appropriate. And first of all, metaphysicians differ a lot from scientists proper. These latter only propose their theory with circumspection; they give it as a first draft capable of being improved; and, if the form of the system, if the building is defective, at least the substance remains, the materials can still be used. The systems of metaphysicians always announce themselves, on the contrary, as being the whole truth, and yet only live for one day; and generally of all the threads of which they are composed, there remains none which can be used to weave a new web. Their vanity has caused them to be compared to soap bubbles or cobwebs, and this is the main cause of the disrepute into which metaphysics has fallen today. There has even been a school, the positive school of which Mr. Comte is the founder, who regards it as the dream of an intelligence barely emerging from childhood; [XIII] what she pursues would only be a phantom created by her. Nonetheless the positivists would yet need to explain the remarkable *fact* that this illusion which has lasted four thousand years, drew Man to search for the solution of a question that had not been asked.

But these attacks, to which metaphysics is subjected, prove only the following: that their authors do not have a fair conception of the object of metaphysics.

We do not dare deny astronomy, because we cannot deny the celestial

[4][Translator's note] By "scientific experience" it is meant experience in the Kantian sense, which is distinct from direct empirical data.

[5]We have used some logical terms, such as *hypotheses, description, genetic definition*, etc., in a sense sometimes a little different from the ordinary sense. We did not think it necessary to motivate these changes because they did not seem to us to bring obscurity to the reasoning.

bodies nor the possibility of explaining their movements. We cannot deny anthropology, seeing that man exists, and that his existence, like all existence, is the subject of a problem. So what is the object, what is the goal of metaphysics?

Science has as its object the universe, but only the universe insofar as it is intelligible. The intelligible universe is the expression revealed to consciousness of a relationship between the self and t,he non-self. It is a series which expresses an unknown function. Is it possible to know the conditions of this relation? This is the problem that the metaphysician poses. We can, like Kant, while recognizing that there is a problem posed, claim that it is unsolvable. [XIV] Metaphysics is thus declared powerless to achieve its goal. What is this goal? It is to explain things (including man and his thought) and their intelligibility by a supreme hypothesis[6]—pantheism, pancosmism, dualism, identity, creation—; and the criterion of the system would be its agreement with the observed facts, assuming established the absolute legitimacy of this criterion.

However incompletely we will have exposed in this work the metaphysical idea which serves as its basis, it is nevertheless clear to see that whilst equally distant from the two extremes, realism and idealism, it seeks to reconcile them both.

The sensualists claim, either, with Hume, that the principles of reason are inductions drawn illegitimately from empirical matter, and that the results of science only ever have a usurped authority and a provisional value; or, with Mill, that induction is a sovereign and incontestable process which, well used, gives empirical observations an obvious character.

On the other hand, the Cartesians think that human intelligence contains in an enveloped state the absolute principles that contact with the world develops. Thus the acorn contains the germ of the oak which the earth, heat and humidity will make blossom. The developed intelligence is, in this way, in harmony with the phenomena. But the action of these is in reality completely inexplicable, to the point that Leibnitz [*sic.*] ends up suppressing it, rubbing shoulders with both subjective idealism and pantheism.

Whatever one does, mysticism is the logical end of this doctrine, as skepticism is at that of the other.

Finally Kant believed he had found the definitive solution to the problem in a conciliation whose true significance did not escape him, and which was fundamentally a reform of science. He claims in fact that intelligence, armed from scratch, imposes its imprint on the external world as perceived

[6]This word being taken in its etymological sense, discussed above, of a *principle that is both subjective and objective.*

by the senses, so that the discoveries of experience are only reflections of the immutable laws of our mind. Kant therefore retained the two parts of the problem; he even admitted their relation; but this relationship, according to him, remains unknown to consciousness. [XVI] By this he recognized that he could not determine the role of this mysterious *noumenon* in the formation of our knowledge; he even forbade himself to talk about it, and, as a result, opened the door again to exclusive solutions which, while remaining on the ground of criticism, destroyed the problem by simplifying it.

Schelling and, after him, Hegel placed the truth in this relationship itself, or in the law governing this relationship. Their famous systems, which today only have a few adherents, have nevertheless brought to light a great truth, which is that the ideal is the only reality; in other words, that everything that is real is rational, that everything that is rational is real. Unfortunately this principle was pushed to the point of abuse; and the result was a scaffolding whose materials were, in large part, only acquisitions of experience, and which were nevertheless given as constructed a priori. And so it immediately crumbled. Should we return to empiricism pure and simple, as has been done in Germany, and, going against the master's doctrine, exclaim that the brute fact contains all ideality? Should we hold metaphysics in contempt and pronounce against it a verdict of sterility and impotence? This is the natural effect of any reaction. Should we not rather, without falling into eclecticism, remember that truth and error almost always exist at the same time in the works of men, that each system has its true side, and that the mark of a good mind is to seek to free it from the falsehood that surrounds it?

It is not surprising, moreover, that, faced with a problem as difficult as that of metaphysics, the finest intellects come to pieces. If, while obeying their rational and free nature which imperiously demands a solution, they have failed, far from casting contempt and ridicule on them, we must admire them for having attempted it. A defeat is not always shameful.

As for us, we have only focused on a secondary issue, which we have attempted to clarify. If we have succeeded, perhaps we will then be allowed to apply the same entirely scientific method to other isolated issues in science. Let us say, however, that we see in intelligence simply a synthetic force which tends towards unity, towards the *uniformization* (if we can risk this word) of phenomena. She feels a need to transform the varied world of *perceptions* into a *universe* of *conceptions*; accidental, fortuitous relationships, into general laws, true everywhere and always; *post hoc into propter hoc*; changing *matter* into invariable *substance*. On one hand, in the world of perceptions, synthetic judgments are the product of a more or less bold, more or less questionable induction; the attribute remains outside the subject, only dresses it, so to speak. On the other hand, in the universe of conceptions, judgments are

analytical; the initially empirical attribute is transformed into an essential character, serving to define the subject.

Might this simple distinction allow us to resolve the problem of antinomies? Perhaps space appears as infinitely divisible, time as having never begun, causes as always being effects of former causes, only in the universe of science, in this universe where everything is defined, labeled, numbered, boxed; whereas the given empirical world would take on completely opposite characteristics, the world of perceptions would purely present the fact, always the fact, without link with what precedes or what follows it, if not a completely fortuitous link of juxtaposition and of inheritance. [XIX] But this is not the place to deal with this vital question of metaphysics, a question much more difficult than that of postulates, to the point that recent philosophies make this antithesis the law of intelligence, the sign of an irreconcilable antagonism between the understanding and reason.[7]

After these few words, which we only wrote to establish the true scope of the second chapter of the first book, all that remains for us is to excuse ourselves for the mixed nature of the subject, if we have at times been too metaphysician for the scientists, too mathematician for the philosophers.

Furthermore, as it is difficult today to know all the works which have appeared on the same question, we will ask the reader's indulgence in the case where, without our knowledge, we have given as our own an idea already expressed. In this manner, we finished our work on the postulates, without suspecting that MM. Erb (1846) and Ueberweg (1851) had written on the same subject, and had anticipated us in some of our critiques, that, among others, of the definition of the plane. [XX] We will, however, draw attention to the collection of our definitions, which we believe are for the most part new, and as precise as possible. We have not concealed our pretensions. We have aimed to give these definitions scientific rigor, and at the same time wanted them to respond to the intuition that we have of the thing defined. This is to give critics a double measure to judge them by.

Before ending we have to say a few words about the translation which is found at the end of the work. Mr. Ueberweg's dissertation, first printed in the Pedagogical Archives, was communicated to us by the author himself, and we made some comments on it. When we decided to have our work printed, we obtained permission from Mr. Ueberweg to translate his, and he was kind enough to write a new introduction where he addressed some of our objections. Likewise in the criticism that we make of this author's

[7] See *la Métaphysique et la Science*, by Et. Vacherot, Paris 1858.

theory, we sometimes refute arguments provided by him, but which were not in the original dissertation. [XXI] The reader will be able to judge these modifications by comparing the two editions of the same work.[8]

On the other hand, we have deleted in the translation, and with the consent of the author, some rigorous but somewhat lengthy demonstrations, which prevented the process and general spirit of the work from being easily understood. We owe it to the truth to say that many of our ideas were awakened by his. We will recognize the same influence in Mill's work on *Inductive Logic*. Finally, allow us to pay here our debt of gratitude to the late A. Meyer, professor of high analysis at the University of Liége, who helped us with his knowledge for the mathematical part, and to the learned professor of philosophy at the same establishment, Mr. Leroy, whose judicious advice was of the greatest use to us, for the plan as well as for the details of our work.

[8]One of the most important is that which relates to the 3rd Experiment. In fact, the author has given up trying to demonstrate that in the body which revolves around two fixed points, there is a continuous line of immobile points.