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Mach's principle and Mach's hypotheses

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ABSTRACT

We argue that the fundamental assertion underlying Mach's critique of Newton's first law is that inertial motion is not motion in the absence of causes; rather, it is motion whose cause lies in some homogeneous aspect of the environment. We distinguish this formal requirement (Mach's principle) from two hypotheses which Mach considers concerning the origin of inertia: that the distant stars play (1) a merely "collateral" or (2) a "fundamental" role in the causal determination of inertial motion.

In his later writings, Mach deliberately avoids referring to the concept of causation, and indeed, this has made the interpretation of Mach's principle a subject of widespread controversy. However, in his earlier writings, the substance of Mach's critique is less ambiguously expressed. Therefore, close attention is given to Mach's early writings and the evolution of his thought. Various accounts in the secondary literature on Mach's principle, in particular those of Norton and DiSalle, are assessed on this basis. We end with a defence of the Machian status and legitimacy of the early Einstein's research program.

1. Introductory remarks

Although established works in the latter half of the 20^{th} Century have painted Mach's contribution to the philosophy of physics in a generally disfavorable light (Bunge, 1966, Blackmore, 1972, Stein, 1977, Earman, 1989), more recent scholarship calls upon us to reconsider this verdict (DiSalle, 2002a, Wolters, 2011a, Banks, 2014, 2012, Thébault, 2021, Staley, 2021). Historically, Mach has always been a divisive figure. In the early 20^{th} Century, Mach inspired the formation of the Vienna Circle which led to the highly influential school of logical positivism, while others such as Lenin (1909) abhorred his reduction of physics to sensation, and slandered his school of thought as a form of "reactionary idealism".

One of the major points of contention concerning Mach to this day is the famous "Mach's principle", a principle about which there is as much disagreement over the interpretation as there are differences in opinion concerning its validity and value. To start with, Mach never defined Mach's principle or even used the term; it was popularised by Einstein, who extracted many diverse and sometimes inconsistent formulations of this idea from Mach's writings. One central source of controversy concerns whether Mach's principle proposes the relational definition of (1) inertial motion, or (2) inertial mass. This question will not be the subject of the present paper, instead, we will take for granted the first interpretation.[1] A second source of controversy concerns the question of whether Mach advocated a "mere redescription" of Newtonian mechanics, as Norton (1995) suggests, or whether he endorsed the development of a new hypothetical law of inertia, as Barbour (1995) argues. While we will side with Barbour, we acknowledge that the controversy here is well-founded. As we shall see, Mach's positivistic skepticism regarding the concept of causation was responsible for both: (1) birthing his highly suggestive critique of the inertial law, and (2) obscuring the distinction between his epistemological concerns and his hypothetical speculations. Mach's reluctance to use explicitly causal language has led to a poorly drawn boundary between what we here call "Mach's principle" and "the Mach hypothesis".

The central aim of this paper is twofold: Firstly, we clarify the distinction between Mach's principle on one hand, which is a purely formal requirement (see section 4.2), and on the other hand Mach's hypotheses, of which two possible classes are considered in his work (see section 4.2.1 and 4.2.2). The second aim of this paper is to show that all of Mach's reflections on the law of inertia are fundamentally informed by his critique of the classical conception of causality according to which singular causes bring about singular effects. The fundamental insight which Mach contributes is the suggestion that inertial motion is not, in fact, *uncaused* motion, but might instead be a motion caused by a roughly homogeneous environment. This is discussed in sections 3 and 4.

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[1] See Barbour (1981) for a detailed justification of this interpretation, and a discussion of Einstein's inconsistent formulations of Mach's principle.

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In later sections of this work, we turn our attention towards some of Mach's more recent critics. Much of the controversy over the interpretation of Mach can be traced back to (1) the failure to distinguish Mach's principle from what we here call the *Mach hypothesis*, and (2) the evolution that the content and style of Mach's thought underwent throughout his life. DiSalle's exemplary scholarship (DiSalle, 2002a, 2002b) shows some awareness of both these issues, and presents a sober defence of Mach which deflects the disparaging attitude of prior critics towards Albert Einstein. We finish the paper with a defence of the Machian status and legitimacy of the early Einstein's research program.

2. The context of Mach

In this section, we will briefly discuss Newton's conception of inertia, his arguments for the existence of absolute space, and the criticism of these by Neumann and Lange, who were contemporaries of Mach. For the purposes of this article, we will not be providing any novel analysis of Newton.[2]

2.1. Newton

2.1.1. Inertia and force

Newtonian mechanics is founded on a paradigmatic distinction between *inert* and *forced* motion. By inducing their accelerations, forces are conceived of as the means by which bodies mutually interact with one another. The concept of inertia makes these interactions intelligible by providing a definition of the motion that bodies resort to in the absence of forces. Newton's first law, the law of inertia, is given as (Cajori, 1934):

> *Every body preserves in its state of rest, or uniform motion in a right line, unless it is compelled to change that state by forces impress'd thereon.*

Newton's second law defines the concept of a force:

> *The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*

If a body moves in a manner which is not uniform and rectilinear, then we must infer that some force, whose cause has its origin in the action of another body (or bodies) upon the first, is responsible for this difference. But this distinction between inert and forced motion requires that we posit some *ground* with respect to which inertial motion is defined. As this ground, Newton postulates the existence of absolute space and absolute time that form the standard with respect to which the rectilinearity and uniformity of inertial motion would be defined.

> I. Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration. [...]
>
> II. Absolute space, in its own nature, without regard to anything external, remains always similar and immovable.

2.1.2. Empirical arguments

In order to demonstrate the existence of absolute space, Newton appeals to the effects of inertial forces, such as the centrifugal force, on rotating bodies. For instance, his famous 'bucket experiment' (see Cajori (1934, p.10-11)), which was devised to undermine the Cartesian idea that relative motions alone exist, relies on this distinction.[3]

Acknowledging the "great difficulty" involved with distinguishing true motions from the apparent, Newton proposes that the observation of forces may help us in this endeavour. As an example of how absolute circular motion might be determined, Newton considers the following thought experiment:

> if two globes, kept at a given distance one from the other by means of a cord that connects them, were revolved about their common centre of gravity, we might, from the tension of the cord, discover the endeavour of the globes to recede from the axis of their motion, and from thence we might compute the quantity of their circular motions.

It is not too difficult to notice however that although certain forms of motion such as circular motion might be deduced by such means, we can not know at what speed we are travelling rectilinearly through absolute space. This is because Newton's laws make no distinction between such cases. Newton, in fact, recognised this and stated it in his well-known Corollary V:

> *The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion.*

In Newton's own time, his conception of absolute space and time was opposed notably by Berkeley, Leibniz and Huygens on the grounds that only relative motions are epistemically accessible.[4] Later commentators who contributed to the relationalist literature included Kant, Laplace and Poisson.[5] The last third of the 19^{th} Century saw a dramatic rebirth of interest in this topic through writers such as Duhamel, Thompson and Tait, Maxwell, Neumann, Lange, Streinitz and of course Ernst Mach.[6] In this paper, apart from Mach, we will briefly examine the contributions of Neumann and Lange, since Mach interacted with these explicitly.

2.2. Neumann's approach

In his inaugural address to the University of Leipzig in 1869, the mathematician Carl Neumann discusses the foundations of mechanics and in particular, raises the question of whether the law of inertia is adequately defined.[7] Neumann remarks that no given body in space is adequate to once and for all determine the definition of inertial motion, since "a motion that appeared to be rectilinear when watched from the earth would appear curvilinear when watched from the sun" (Neumann, 1993). The true content of Newton's reference to 'absolute space' for the definition of rectilinearity consists in the assertion that all motion must be referred to the *same* object:

> For the character, the essence of the so-called absolute motion consists (as no one can deny) in that all changes of place are referred to one and the same object — namely an object that is extended and unchanged although it cannot be assigned more concretely.

[2] See Guicciardini (2018) for an extensive discussion of Newton's views and references to further literature.

[3] According to Descartes, the motion of the water should be defined in relation to the sides of the bucket, but this motion, Newton shows, has no bearing on whether the water recedes up the sides of its container. Newton's disagreements with Descartes are expressed more explicitly in his unpublished essay *de gravitatione* (Newton, 1962). See Barbour (2001, p.609-624) for a detailed discussion of this work.

[4] For Berkeley's opposition see Berkeley (1999/1710, para. 112-117) and Berkeley (1992/1721, para. 59-60); Leibniz's commentaries on the issue can be found for instance in Leibniz (2000/1716), Leibniz (1989b, p.308) and Clarke (1717); for the relevant comments from Huygens, see Darrigol (2021).

[5] See Kant (1970/1786), Laplace (1796) and Poisson (1811) respectively.

[6] These discussions can be found in the following texts: Duhamel (1870), Thomson and Tait (1867), Maxwell (1892/1876), Neumann (1993), Lange (2014/1885), Streintz (1883). For a thorough overview of these contributions, see Darrigol (2021, chp.3).

[7] For further discussions of Neumann's contribution, see Thébault (2021), Darrigol (2021), Pulte (2009).

For this reason, Neumann introduces the notion of a hypothetical "*Body Alpha*", which specifies three coordinate axes, with respect to which rectilinear motion should be defined: "a material point left to itself proceeds in a straight line—i.e., in a path that is rectilinear in relation to this Body Alpha." Neumann insists that this Body Alpha has just as much right to exist as the "luminiferous ether or the electrical fluid", since it is necessary as an object with respect to which inertial motion is to be defined. Thus Neumann presents three principles of Galilean-Newtonian theory which are given as follows:

1. "in some unknown position in space an unknown body exists, and indeed an absolutely rigid body, a body whose figure and dimensions will be immutable for all time."
2. "a material point left to itself proceeds in a straight line — i.e., in a path that is rectilinear in relation to this Body Alpha."
3. "Two material points, each of them left to itself, move in such a way that the equal paths of one of them always correspond to the equal paths of the other."

The first two principles have to do with the definition of *rectilinearity* in Newton's first law; the Body Alpha is needed to define the rigid coordinate axes with respect to which particles move inertially. The third principle concerns the question of the *uniformity* of the motion. Neumann does not suppose the existence of an *Alpha Clock*, but rather requires that any two material points in inertial motion must move uniformly with respect to one another. It was on this basis that Lange would criticise Neumann's solution to the problem of defining the inertial law.

2.3. Lange's approach

In his paper of 1884, titled *On the law of inertia* (Lange, 2014/1885), Lange proposes a way by which it may be possible to solve this epistemological difficulty while dispensing with the need of absolute space or any substitute such as the 'Body Alpha' of Neumann. Lange remarks that in Neumann's third principle the uniformity of the motions of bodies is defined in terms of the mutual consistency in the behaviour of bodies, without the need to postulate some absolute structure.

> We already have a fully valid substitute for the absolute time. [...] We have only, following Neumann, to base the measure of time on the following definition: Two time intervals are said to be equal in which a point left to itself passes through equal spatial distances. [...] Under this viewpoint, the law of the "uniform" motion of all points left to themselves is, as Thomson and Tait correctly note, a pure convention for one such point, and it is more than convention, it is a research result, only insofar as it applies to any other points left to themselves.

What Lange sets out to do then, is to extend Neumann's reasoning to the definition of rectilinearity in the inertial law:

> The question now arises whether it is possible to eliminate also absolute space by a similar procedure. Indeed this is possible.

Lange observes that for any three given points in an empty space that move arbitrarily, "it is always possible to construct a coordinate system, indeed infinitely many coordinate systems, in relation to which these points move rectilinearly." On the other hand, if more than three points are considered, this is only the case "under special circumstances, only contingently." Just as Thomson and Tait (1867, para.247-248) had formerly remarked that the uniform motion of a single material point is a *convention*, Lange now asserts that the rectilinearity of the motion of three material points is purely conventional. The true empirical content of the law of inertia thus consists in asserting that given three material points whose motion defines an inertial system, any fourth material point must move rectilinearly and uniformly with respect to the inertial system defined by the first three. In this way, by Applying Neumman's reasoning in his third law to the definition of rectilinearity, Lange claims to have disposed with the difficulty regarding the definition of inertial motion, the Body Alpha is not needed, and neither is absolute space; particles simply travel rectilinearly and uniformly with respect to one another.

3. Causality and determination

In this section we will bring attention to two issues concerning the concept of causal determination which are critical to understanding the different aspects of Mach's critique of Newton's first law. As we will see, Mach was aware of both these issues, however this is not always made clear in his writings, therefore it is important that we understand these issues clearly before moving on to interpreting Mach's critique.

3.1. Two issues

Issue 1. Distinction between epistemological and causal determination.

At the heart of the issue raised at the end of the last section is an ambiguity concerning the definition of the word '*determine*'. In the first place there is an epistemological question: '*how can I determine the truth of this statement?*' Here the word 'determine' refers to the means by which my knowledge of something is reached. On the other hand, there is a question pertaining to physical causality: '*what is it that determines the motion of this body?*' Here the word 'determine' refers to the physical determination of the future state of a system by its prior state in accordance with the law of causality. Neumann's solution to the problem of the determination of inertial motion involves a conflation between these two. By introducing the 'Body Alpha', Neumann implicitly supposes that it is necessary to consider a cause of the rectilinearity of inertial motion,[8] but neglects the task of describing this in terms of objects which we have epistemological access to. On the other hand, in his analysis of the uniformity of inertial motion, we find the converse: Neumann ignores the need for such a causal determination, and focuses only on the task of describing this motion in terms of facts which we have epistemological access to. Lange, on the other hand, is not at all concerned with the postulation of real objects that would be responsible for causally determining inertial motion, but only with distinguishing the empirical content of the law from the conventional part. In this way, Lange leaves unanswered the question of causation but manages to express the content of Newton's law without reference to unobservable structures.

Issue 2. Neglect of stable environmental factors in classical accounts of causation.

The second issue concerning the notion of causality is particularly salient to Mach's critique of Newton. This is the idea that changes in the state of a system are not brought about by singular, isolated causes, but rather, it is the entire state of a given system that is responsible for bringing about the subsequent state. It is only our habituation with systems whose environment is relatively stable that leads us to assume that environmental influences can be ignored.

Both of these issues concerning the concept of causation are addressed by Mach in his fifth appendix to the second edition of *Die Mechanik* (1883). Concerning the first issue, Mach writes (Mach, 1893/1887, p.516):

[8] Indeed Neumann explicitly acknowledges inertial motion as caused motion (Neumann, 1993, p.359): "*Inertia* exists and, simultaneously, the *attraction* of the earth exists as well. In consequence of the combination of *both* these causes, arises the motion in which the stone traverses the parabolically curved path" (original emphasis).

> In the text I have employed the term "cause" in the sense in which it is ordinarily used. I may add that with Dr. Carus, following the practice of the German philosophers, I distinguish cause, or *Real-grund*, from *Erkenntnissgrund*.[9]

In his essay *History and Root of the Principle of Conservation of Energy* (henceforth *Conservation of Energy*), initially published in 1872, Mach discusses this second issue in some depth. Throughout the text, Mach traces the many forms of the principle of conservation of energy historically to their derivation from some requirement for the exclusion of perpetual motion. In the final chapter, titled *The Logical Root of the Theorem of Excluded Perpetual Motion*, Mach traces this theorem back to the *law of causality*. Mach emphasises that this law does not refer single causes to single effects, but rather (Mach, 2014/1872, p.63):

> the totality of the phenomena on which a phenomenon α can be considered as dependent, [should define] *the cause*.

Thereby Mach expresses the law of causality as: "the effect is determined by the cause" ("die Wirkung ist durch die Ursache bestimmt").

3.2. Archimedes' proof and Mach's changing language

To illustrate the colloquial misapplication of the causal law, or of a similar law, that of 'sufficient reason', Mach recounts Archimedes's justification of the equilibrium of a scale which on both sides holds equal weight. Archimedes explains the observed lack of motion in terms of the absence of any *reason* why the bar should "turn in one direction rather than in the other." Mach insists however that this is not expressed correctly[10]:

> Only this is not expressed quite properly: we ought rather to say that there is a reason that, in these cases, nothing happens. For the effect is determined by the cause, and the one and only effect which is here determined by the cause is no effect at all. (Mach, 2014/1872, p.66)

By the time he publishes *Die Mechanik* in 1883, Mach's view seems to have changed a little. We have already mentioned his note from the appendix in which he agrees with Carus's characterisation of cause and effect as "to a great extent arbitrary" (Mach, 1893/1887, p.516). This view is more elaborately expressed in chp. IV, sec. IV, para. 3:

> In speaking of cause and effect we arbitrarily give relief to those elements to whose connection we have to attend in the reproduction of a fact in the respect in which it is important to us. There is no cause nor effect in nature; nature has but an individual existence; nature simply is.[11] (Mach, 1893/1887, p.483)

Mach goes on to discuss the views on causality of Hume, Kant and Schopenhauer, and situates himself closest to that of Hume. To make it even more clear that Mach's thinking concerning causality had evolved since 1872, we need only look as far as chp. I, sec. I, para. 2 of *Die Mechanik*, in which Mach discusses exactly the same justification for the equilibrium of a scale by Archimedes as he had discussed in *Conservation of Energy*. In his treatment of the issue now however, Mach makes absolutely no mention of the word 'cause', although, just as in *Conservation of Energy*, Mach directs his criticism towards the neglect of the surrounding circumstances (Mach, 1893/1883, p.9):

> We might suppose that this [equilibrium] was self-evident entirely apart from any experience, agreeably to the so-called principle of sufficient reason; that in view of the symmetry of the entire arrangement there is no reason why rotation should occur in the one direction rather than in the other. But we forget, in this, that a great multitude of negative and positive experiences are implicitly contained in our assumption.

The "negative" experiences referred to here are the circumstances which experience has taught us play no role in the determination of the equilibrium, such as the "colours of the lever-arms" or "the position of the spectator". The "positive" experiences on the other hand are the other neglected determinative circumstances such as the distances of the weights from the supporting point. Mach then concludes that:

> By the aid of these experiences we do indeed perceive that rest (no motion) is the only motion which can be uniquely determined, or defined, by the determinative conditions of the case.[12] (Mach, 1893/1883, p.10)

Which is an analogous conclusion to that which Mach pronounced in 1872: "the effect is determined by the cause, and the one and only effect which is here determined by the cause is no effect at all" (Mach, 2014/1872, p.66), except that by 1883 Mach has dropped any reference to the concept of causation, but chooses instead only to use the word "determined".

It is clear from this comparison to his earlier work that much of the content of Mach's views on causality has not changed; he is still chiefly concerned about the neglect of environmental circumstances ("negative and positive experiences"), in our colloquial use of sufficient reason or causality. However, by the time he comes to write *Die Mechanik*, Mach more fully embodies his mature positivist style, and therefore shows a much greater reluctance to use the word "cause". This point should be kept in mind when we move on to analysing Mach's critique of the law of inertia, in particular, we should remember that when Mach refers to the notion that the motion of a body is *determined* by such and such a circumstance, he is not merely making an epistemological claim. Rather, he is referring to that which in 1872 he would have called a *cause* (i.e. "*Ursache*" or "*Real-grund*") of the phenomenon.

4. Young Mach: his principle and hypotheses

In the context of 19^{th} Century discussions of *force*, the following two quotes by Poisson and Maxwell give us a good impression of the generally accepted view. Poisson (1811) expresses himself as follows:

> if we consider a body at the instant it passes from a state of rest to a state of motion, we may always observe, that this change is owing to the action of an extraneous cause, [...] Any cause which excites motion in a body, [...] is called force.

Similarly, Maxwell (1892/1876) defines a force as:

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[9] Mach's reference to "the practice of German philosophers" is quite appropriate since what Mach is saying concerning the law of causality was well understood by many philosophers in his day. For instance, as early as 1813, Schopenhauer discussed the very same two issues in his doctoral dissertation (Schopenhauer, 1997/1813).

[10] This passage also demonstrates that Mach had a broader, less anthropomorphic conception of causality than Leibniz, since Leibniz raises no objections to Archimedes' use of the principle of sufficient reason in his own treatment of the passage (Clarke, 1717, para. 1). As we will argue, it is this very departure from the traditional anthropomorphic notions of force and inertia that enables Mach, unlike Leibniz, to develop a relationalist resolution of the problem of inertial forces.

[11] The words used for "cause and effect" in the original German text are: "Ursache und Wirkung".

[12] Here the German word which is translated as "determined" is "bestimmte".

> in every case in which we find an alteration of the state of motion of a body, we can trace this alteration to some action between that body and another, that is to say, to an external force.

But for a 'change' or 'alteration' to be able to take place, there must be some standard by which motion will be determined in the absence of such changes. By necessity therefore, we require some '*law of inertia*'.

Now if we apply Mach's criticism of the neglect of environmental circumstances (exemplified by his treatment of Archimedes' proof) to the traditional conception of *force*, we find that we will need to call into question this paradigmatic distinction between inertial and forced motion. Are we truly ever justified in isolating the influence of a single body on another? Who is to say what would happen to the two bodies if the rest of the world did not exist?[13] By singling out the effect of a single body and regarding the rest as negligible, we commit an epistemological *sin*: the neglect of the stable background environment, which we implicitly assume can play no part in the causal determination of the motion considered. Mach therefore proposes that the inertial law does not define motion in the absence of causes, but rather represents the motion caused by the stable background environment constituted of the distant masses of the universe.

If the *variability* in motion of a body can be traced, as Maxwell states, to a corresponding change in another body, might we suppose that the *invariance* of inertial motion, which, after all, we define epistemically w.r.t. the fixed stars, is a consequence of the relative invariance of this same cosmic environment? This is the intuition that underlies Mach's famous hypothesis. In this way, it is due to his enlarged conception of causation, his insistence that the *environment* cannot be neglected when we consider the cause of a given motion (issue 2), that Mach hypothesises the action of the distant stars in the determination of inertial motion. This becomes apparent in his reflections shortly following his discussion of the hypothesis in *Die Mechanik*[14]:

> The most important result of our reflexions is, however, *that precisely the apparently simplest mechanical principles are of a very complicated character, that these principles are founded on uncompleted experiences, nay on experiences that never can be fully completed, that practically, indeed, they are sufficiently secured, in view of the tolerable stability of our environment.* (Mach, 1893/1883, p.237)

By referring to "uncompleted experiences" ("unabgeschlossenen [...] Erfahrungen"), Mach means to say that our experiences of the phenomenon of inertia does not include both sides of the relation. Since absolute space is not an object of knowledge, whatever real thing it stands for in our mechanics has not been sufficiently mapped out by our experience.

The interpretation of Mach's views given so far follows logically from the above discussion of his views on causality expressed in his early writings. However, the interpretation of Mach is a contentious issue; moreover, DiSalle (2002a) and Thébault (2021) in particular have raised the importance of placing Mach's writings in the context of the evolution that his thought underwent throughout his life. As Thébault (2021) points out, there is a split in the secondary literature concerning Mach's critique of inertia, where DiSalle and Norton (1995) argue that Mach is "proposing a *redescription* of Newtonian mechanics without absolute concepts", while Barbour (1995) "takes Mach to be proposing a *new theory of inertia*". Indeed, Norton (1995) traces this split in opinion all the way back to the time of Mach's own writings, at which time Paul Carus endorsed the "mere redescription" interpretation, opposing himself to the interpretations of Philip Frank and others. In this paper, we defend an interpretation of Mach that aligns more with Barbour's; the opposing view of Norton will be addressed in section 4.2, while the somewhat more subtle account given by DiSalle will be examined in section 6.

4.1. *Mach's principle: inertia in rotating frames*

Unlike Neumann and other commentators on the law of inertia who took for granted the empirical validity of this law, Mach was the first to seriously raise the question of whether this law might only be right by approximation. Mach's first published comments on this issue appeared in the *Notes* to his formerly discussed *Conservation of Energy* (Mach, 2014/1872). Having addressed this issue prior to Neumann in a series of lectures in 1868, Mach felt emboldened to publish his thoughts in print following Neumann's inaugural address of 1870. Although Mach finds that he and Neumann had identified exactly the same difficulties with the definition of the inertial law, he insists that their solutions differ.

The difficulty which Mach refers to is expressed as follows (Mach, 2014/1872, p.76-77):

> Obviously it does not matter whether we think of the earth as turning round on its axis, or at rest while the celestial bodies revolve round it. Geometrically these are exactly the same case of a relative rotation of the earth and of the celestial bodies with respect to one another. Only, the first representation is astronomically more convenient and simpler. But if we think of the earth at rest and the other celestial bodies revolving round it, there is no flattening of the earth, no Foucault's experiment, and so on—at least according to our usual conception of the law of inertia. [...] The law of inertia must be so conceived that exactly the same thing results from the second supposition as from the first.

About a decade later, in his first edition to *Die Mechanik* (1883), Mach expresses the same concern (Mach, 1893/1883, p.232):

> Relatively, [...] the motions of the universe are the same whether we adopt the Ptolemaic or the Copernican mode of view. Both views are, indeed, equally correct; only the latter is more simple and more practical.[15] [...] The universe is not twice given, with an earth at rest and an earth in motion; but only once, with its relative motions, alone determinable. It is, accordingly, not permitted us to say how things would be if the earth did not rotate. We may interpret the one case that is given us, in different ways. If, however, we so interpret it that we come into conflict with experience, our interpretation is simply wrong. The principles of mechanics can, indeed, be so conceived, that even for relative rotations centrifugal forces arise.

In both cases, we see that Mach is asserting a *kinematical* equivalence between two situations, and insisting that our laws be expressed in such a way that the same *dynamical* evolution be observed. In *Die Mechanik*, which is written a decade later than the first, Mach shows more caution, emphasising that if experience were shown to be inconsistent with the Ptolemaic picture, the imposition of an equivalence ought to be abandoned. However, Mach subsequently aims to show that it is possible to formulate the laws of mechanics in such a way that the rotation of our frame becomes a genuine symmetry of the system.[16]

[13] Interestingly, Poincaré raises very similar questions concerning the definitions of forces and inertial motion in Poincaré (1905/1902, chp.VI).

[14] Emphasis in original.

[15] It is worth noting that the equivalence of the Copernican and Ptolemaic perspective had already been recognised by Leibniz in his essay of 1689 *On Copernicanism and the Relativity of Motion* (Leibniz, 1989a) who, like Mach, appealed to the notion of simplicity in defence of the Copernican view.

[16] As Reichenbach (1965, p.8) remarks, the need to generalise the relativity of motion to rotating frames of reference is a natural consequence of the Kantian conception of space. Although Mach approached the question from a strictly empiricist standpoint, it is worth remarking that Mach was deeply influenced

Since this equivalence is only a *formal* requirement, and does not place constraints upon the possible content of our physical laws, we cannot call it a *hypothesis*. Rather, we may call it '*Mach's principle*', and define it as follows:

Principle 1. The dynamical evolution of an isolated system depends only upon its relational kinematical configuration.

In other words, it is the observable relations between the parts of a system that have causal power, not the values of certain unobservable absolutes.[17]

The immediate corollary of this principle is that the laws of physics ought to be invariant under transformations of our coordinates that keep the relational configuration of real things invariant, as Mach recognises.

Corollary 1.1. *The laws of physics ought to take the same form under any reference frame related to an inertial frame by a rigid transformation.*

Since this principle does not specify the content of this relational kinematical configuration, it in no way constrains the possible content of physical laws. For instance, Newton's mechanics may be saved if we simply regard "absolute space" as a physical entity distinct from geometrical space, or else imagine that Neumann's privileged 'Body Alpha' exists.

4.2. Mach's hypotheses

The problem which Mach had identified in the definition of the law of inertia does not lead directly to Mach's hypothesis about the fixed stars, in both *Conservation of Energy* and *Die Mechanik*, Mach is more cautious than to claim this. In *Conservation of Energy*, Mach identifies two possible solutions to the problem:

1. That which Neumann prefers: that "all motion is absolute," or referred to some hypothetical 'Body Alpha'.
2. That the law of inertia is wrongly expressed, and in particular that "in its expression, regard must be paid to the masses of the universe."

In the first edition of *Die Mechanik* (1883), we find a similar choice of two possible approaches to the problem, one which would secure Newton's law and another which would imply an alternative formulation. These will each be discussed in what follows.

4.2.1. First hypothesis: stars as collateral

The first possible solution to the problem of the origin of inertial motion is presented in *Die Mechanik* prior to the passage quoted above. After directing a purely epistemological critique at Newton's formulation of the law of inertia, by insisting that the empirical basis of our knowledge of inertial motion always consists of a relative motion between the given body K and the other bodies in the universe A, B, C, ..., Mach proposes a possible solution to the problem (Mach, 1893/1883, p.230-231):

> It might be, indeed, that the isolated bodies A, B, C. ... play merely a collateral role in the determination of the motion of the body K, and that this motion is determined by a *medium* in which K exists. In such a case we should have to substitute this medium for Newton's absolute space. [...] In itself such a state of things would not belong to the impossibilities.

Although the transition here from a question of epistemological determination to a question of causal determination is not made explicit as Mach again avoids using the word 'cause', we can safely assume that he is referring to a causal determination or '*Real-Grund*', since the terms "determined" ("bestimmt wäre") is here used in a very similar context as his discussion of Archimedes's proof. Furthermore, if Mach were merely concerned with providing an account of our epistemological determination of inertial frames, he would have no reason to appeal to the hypothetical existence of this "medium", to which we do not even have direct epistemological access.[18]

Instead of the absolute position and velocity of bodies being responsible for *causally* determining their evolution, Mach proposes that this may be determined by a form of relative motion and position; a motion relative to a hypothetical medium. By considering this hypothesis Mach demonstrates that he is not so naive as to think that inertial motion must be causally determined by some relation between objects which we already have direct epistemological access to, but rather, that this motion should be determined by some relations which are *in principle* epistemically accessible. Indeed Mach goes on to praise the potential fruitfulness of this hypothesis if its pursuit might allow us to discover the other physical properties of this medium.

Hypothesis 4.1. The motion of bodies observed as rectilinear and uniform with reference to the fixed stars, which is identified as 'inert' in classical mechanics might be caused by the action of a medium in which these bodies and the stars are embedded.

In later editions of *Die Mechanik*, Mach likens this *medium* solution to that of Budde (1890, p.133-136), who, like Mach, was troubled by the idea of having a preferred class of reference frames without these being determined by some relation to a real thing. Budde postulates that space is some kind of medium which is responsible for determining the motion of the bodies contained. Mach simply adds to this in his appendix (Mach, 1893/1887, p.547)[19]:

> I have no objections to Budde's conception of space as a sort of medium (compare page 230), although I think that the properties of this medium should be demonstrable physically in some other manner, and that they should not be assumed *ad hoc*.

4.2.2. Second hypothesis: stars as fundamental

Having covered the possibility of the existence of this medium, Mach puts aside this particular hypothesis in order to consider his own favoured hypothesis according to which a relation to the distant stars would play a "fundamental" ("wesentliche") role rather than a "collateral" ("zufällige") role in the determination of inertial motion. We can take this to mean that Mach would now like to consider the hypothesis that the fixed stars are responsible for the *causal* determination of the motion of inertial bodies rather than merely being our means of gaining knowledge of inertial frames (determined by the hypothetical

by Kant, in particular by Kant (2004/1783); see Mach (1890, p.65-66) and Mach (2014/1872, p.16).

[17] This type of formulation of Mach's principle was recognised by Barbour and Bertotti (1982) who drew from Poincaré's reflection in (Poincaré & Halsted, 2015/1913, p.83-85, p.107-114) to formulate a more precise expression of Mach's principle. For a discussion of Poincaré's contribution, see Mercati (2018).

[18] This in itself is enough to show that the view of Norton (1995) according to which Mach is proposing a "mere redescription" of Newton's mechanics, and is not concerned with identifying a material cause of inertial motion, is mistaken. We will return to this in section 4.2.3.

[19] It is worth noting that Mach's contemplation of this *medium* solution lends credibility to accounts by Weyl (1922, p.218-219) and Brown (2005, p.159-160) according to which general relativity may be regarded as Machian since the existence of this "medium", i.e. *space-time*, is indeed demonstrably by other means (through phenomena of gravitation).

medium).[20] To illustrate how this might work, Mach considers motion in a non-inertial frame of reference. As Newton observed, it is necessary to introduce inertial forces (centrifugal and Coriolis) in this frame if we want to recover an empirically adequate description of the dynamics. But *where* do these inertial forces come from? For Newton, these forces are *fictitious*, they are not real, but rather they are simply an artefact of our choice of reference frame that is rotating in absolute space. According to the hypothesis of a physical medium, or likewise Neumann's hypothesis of the 'Body Alpha', if this body/medium is assumed to be causally responsible for determining inertial motion, we may suppose that these inertial forces are produced by a relative rotation with respect to the body or medium. But what if we want to describe inertial forces in the absence of this physical medium, the Body Alpha, or of absolute space? In this case, Mach proposes that the inertial forces might be (Mach, 1893/1883, p.232):

> produced by [a] relative motion with respect to the mass of the earth and the other celestial bodies.[21]

Similarly, if we were to choose a frame of reference in which no inertial forces are required, it would likewise be the same "celestial bodies" (which would in this case be approximately stationary) that are responsible for causing the rectilinear and uniform motion of a given body that would be observed. In both cases however, what is important is that inertial motion is wholly determined by a body's relations to other physical bodies, and these relations are independent of whether we choose our frame to rotate.

Hypothesis 4.2. The Mach Hypothesis: The motion of bodies observed as rectilinear and uniform with reference to the fixed stars, referred to in classical mechanics as 'inert', might be caused directly by the action of these distant stars upon those bodies.

In paragraph 7 of this same section, Mach goes on to consider possible ways of formulating such a relational replacement of Newton's law of inertia:

> Instead of saying, the direction and velocity of a mass μ in space remain constant, we may also employ the expression, the mean acceleration of the mass μ with respect to the masses m, m', m''. ... at the distances r, r', r''. ... is $=0$, or $d^2(\sum mr/\sum m)/dt^2=0$. The latter expression is equivalent to the former, as soon as we take into consideration a sufficient number of sufficiently distant and sufficiently large masses.

Accordingly, Newton's law of inertia would not be true absolutely, but only true approximately provided that the universe is populated by a sufficiently large number of masses. Unlike Lange and Neumann, Mach is genuinely raising the possibility of an alternative physical law. Mach emphasises however, that this alternative formulation should not be taken as a definitive replacement of the law of inertia, rather, it is simply intended as a demonstration that it is possible to construct a (so far) empirically adequate relational account of inertia as the action of the distant stars. However, with his empiricist humility, Mach leaves the task of developing a more precise replacement for the law of inertia to future investigators:

> It is impossible to say whether the new expression would still represent the true condition of things if the stars were to perform rapid movements among one another. The general experience cannot be constructed from the particular case given us. We must, on the contrary, wait until such an experience presents itself. Perhaps when our physico-astronomical knowledge has been extended, it will be offered somewhere in celestial space, where more violent and complicated motions take place than in our environment.

4.2.3. Norton's interpretation

It is interesting to remark here, that while we have taken the last two passages quoted as clear evidence for the idea that Mach is exploring the possibility of a new physical law (in accordance with Hypothesis 4.2), Norton (1995) cites the very same passages as purported evidence that Mach is proposing a mere "redescription" of Newton's law. Although from the perspective developed so far, this view seems rather strange, it can be made to seem somewhat more credible if we follow Norton's reasoning closely. From Norton's perspective, the proposal of an alternative expression $d^2(\sum mr/\sum m)/dt^2=0$ is merely an attempt at an imperfect redescription of the Newtonian inertial law (Norton, 1995):

> The project is clearly just one of redescriptions of existing laws and not the proposal of a new mechanism. Indeed Mach soon makes it very clear that his new expression for the principle of inertia is not intended to be applied to cases remote from experience.

Norton goes on to cite the second passage quoted above: "It is impossible to say whether the new expression would still represent the true condition of things if the stars were to perform rapid movements among one another [...]". To Norton, this latter qualification is an admission by Mach that his relational law is imperfect since it would fail to adequately approximate the true law of inertia—that of Newton—in a more violent astronomical environment. In other words, Norton understands the term "true conditions of things" as a reference to Newton's first law. However, this reading is, in fact, inconsistent with what is said; since it is already clear that the relational expression contradicts Newton's law, and no empirical scrutiny is needed to prove that. Empirical results will instead be able to tell us *which* expression, be it Mach's proposal, Newton's law, or some other as yet unknown expression, is correct.

Norton is not alone, however, in his interpretation of Mach. Indeed there is a long tradition of interpreting Mach's comments as proposing a mere redescription of Newton's law in terms of observable relations which stretches back to Paul Carus. The reason for this ambiguity in the interpretation of Mach's writings is of course Mach's reluctance to use what he saw as metaphysically loaded, causal language. This reluctance is less pronounced in his earliest writings (Mach, 2014/1872), in which we see the germs of his sceptical attitude towards the common notion of cause and effect articulated more clearly.

5. Mach's critics

5.1. Early Mach's excesses

In his *Notes* from *Conservation of Energy*, Mach expresses his pleasure to have discovered that Neumann shared his concern regarding the definition of the law of inertia:

> Although I was sorry to have lost the priority in this important matter, yet the exact coincidence of my views with those of so distinguished a mathematician gave me great pleasure and richly compensated me for the disdain and surprise which almost all the physicists with whom I discussed this subject showed. (Mach, 2014/1872, p.76)

Over the course of the 20^{th} Century, Mach's views gained a lot more popularity, especially due to Einstein, who coined the term 'Mach's principle' and marketed this idea as a fundamental insight that contributed towards the development of his theory of general relativity.

[20] We may also note that Mach's use of the terms "fundamental" and "collateral" here is due to his deliberate avoidance of using more metaphysically loaded language related to the concept of causation.

[21] Here the verb "produced" is a translation of "geweckt werden", which can also be translated as "awoken".

Mach's principle, and Mach's philosophy in general became associated with logical positivists such as Reichenbach and Schlick, who played a leading role in the philosophical interpretation of Einstein's relativity theories. In the latter half of the 20^{th} Century however, the influential works of a more realist school of philosophers of physics, mostly coming out of Chicago (Bunge, 1966, Stein, 1977, Earman, 1989, Friedman, 2014/1983) painted Mach's contribution in a much less favourable light. Stein in particular launches an aggressive rhetorical attack on Mach, characterising his style of thinking as a form of "abusive empiricism" and going on to dismiss each of his arguments and claims. Stein's attack is reminiscent of the "disdain and surprise" that Mach recounts as the typical response to his ideas from his contemporaries. Although Stein is excessively dismissive, there is a sense in which we can agree with him that, especially in his early writings, Mach's empiricism overlooks the significance of Newton's methods, failing to appreciate what is of value in the postulation of absolute space and time.

After quoting Newton's views on space, time and inertia in *Die Mechanik*, Mach immediately jumps to accusing Newton of acting "contrary to his expressed intention only to investigate *actual facts*" on the basis that absolute space is not observable (Mach, 1893/1883, p.229). But this ignores the methodological significance of the concept of absolute space, which had been recognised by other commentators prior to Mach.[22]

5.2. Late Mach's doubts

In later editions of *Die Mechanik*, Mach seems to soften his criticism of Newton. This appears to be stimulated in particular by his reading of Lange (2014/1885), who's investigation Mach greatly admires for its "methodical movement" which "wins at once the reader's sympathy." It is somewhat peculiar that Mach appreciates Lange's piece so much, given that, as we have seen Lange does not even attempt to solve the *causal* question concerning the determination of inertial motion. For this reason, Mach's admiration of Lange's works raises doubts as to whether Mach even understood his own arguments. Furthermore, in these later editions Mach seems to revisit Newton's ideas from a refreshed perspective, citing Newton's Corollary V (the Galilean principle of relativity) as a formulation of the inertial law which is consistent with Mach's views.

> In order to have a generally valid system of reference, Newton ventured Corollary V of the Principia. He thought of a [...] coordinate system for which the law of inertia holds, fixed in space without rotation relative to the fixed stars. He could also allow an arbitrary origin and uniform translation of this system [...] without loosing its usefulness. Newton's laws of force would not be thereby altered; only the initial position and velocity, and the constants of integration could vary. By this formulation, Newton specified *precisely* the meaning of his *hypothetical* extension of the Galilean law of inertia. One can see that the reduction to absolute space was in no way necessary, since the reference system is no less relatively determined as in any other case. (Mach, 1933, p.227)

Stein interprets this difference to Mach's other views as an indication of Mach's confusion and inconsistency: "This point of view is precisely the appropriate one for Newtonian dynamics; and it rests, as Mach entirely fails to notice, not indeed upon absolute space, but nonetheless upon "absolute uniform motion" as a *vera causa*—not explicated through phenomena of relative motion" (Stein, 1977).

Stein's motives for giving this highly rhetorical account should be questioned. As Banks (2014) and Wolters (2011b) have remarked, the "second wave of Mach scholarship"[23] to which Stein belongs, was "instrumentalized for a battle against the institutionalized positivism" (Banks, 2014, p.9). Other authors have suggested that the apparent inconsistencies in Mach's views indicate an evolution in his thinking (DiSalle, 1990, 2002a, 2002b, Thébault, 2021). It is certainly plausible that Mach's views evolved due to the inherent tension among his ideas. One such tension, which Norton identifies, is between the requirement for economy and the need to reduce theoretical entities to experience (Barbour & Pfister, 1995, p.56):

> There was a tension [in Mach's writings] between the need for the descriptions to be restricted to observation and for them to be economical. [...] the price of the economy is talk of entities that transcend observation. So it is with spacetime structures; they are unobserved, but, [...] they do enable just the systematization we want.

Another tension in Mach's views is that between his strict empiricism and the aprioristic implications of his principle. Moritz Schlick in particular was keen to point this out (Schlick, 1915). For Schlick, Mach's assertion that two kinematically equivalent situations ought not to differ dynamically is insufficiently empiricist since it represents an a priori restriction on our possible theories. In Newton's theory, for instance, two kinematically identical situations may differ dynamically, but this is not a problem, because the difference is accessible to sense-experience (Schlick, 1915, p.168):

> We can also ascertain the absolute rotation of a body, according to the Newtonian view, through muscular sensation, for we will find with its help that centripetal forces are needed for the body to keep its shape and to hold together its parts.

Although this Machian requirement does not in fact restrict the possible empirical content of theories, but only informs their formal aspect, it is plausible that, as an avowed empiricist, Mach felt unsettled by his own ideas. After all, the history of 20^{th} Century physics has produced many examples, whatever one may think of their plausibility, of broad and far-reaching aprioristic speculations based off of Mach's hypothesis.[24] The view that Mach was disturbed by his own ideas is supported by the account of Hugo Dingler who alleges that Mach's explanation of centrifugal forces in terms of a relation to the fixed stars "contradicted his sensibilities" (Dingler, 1921, p.157). Dingler also cites Mach's son, Ludwig Mach, who claims that his father was tormented by the consequences of his hypothesis (Dingler, 1921).[25]

It is quite plausible that these tensions in Mach's thought may explain why he repeatedly feels the need to fall back on cautious epistemological remarks in his treatment of the inertial law, and refrains from further developing his hypothetical speculations. After all, unlike his 20^{th} Century acolytes, Mach did not embark very far upon the project of constructing a new hypothetical physical model that would embody his ideas. Towards the end of his life, he seemed determined to take up the defensible standpoint of a humble empiricist.

6. DiSalle and Einstein: spacetime as constraint

While Stein saw the contradiction between Mach's ideas as evidence of his confusion, DiSalle on the other hand presents a more sympathetic

[22] See for instance Laplace (1796, chp.1) or Kant (1970/1786, p.16).

[23] This term originates in Blackmore (1972).

[24] These include: Hofmann (1995), Reissner (1995), Schrödinger (1925), Sciama (1953a, 1953b), Brans and Dicke (1961), Barbour and Bertotti (1982), Assis (1989) to name just a few examples other than Einstein's project of course.

[25] Dingler's account should be taken with a certain degree of skepticism since, as Norton (1995) remarks, "by 1921, Dingler had become an outspoken critic of relativity theory and, as a disciple of Mach, may well have been overeager to seek reasons to remove Mach's support from relativity theory" (Norton, 1995). Moreover, Gereon Wolters has raised serious doubts concerning the reliability of Ludwig Mach's accounts of his father (Wolters, 2019, 2011a, 1987), so this too should be viewed with skepticism.

interpretation which takes these passages as evidence of an evolution in Mach's thinking: "in accepting the abstract formulation of the laws of motion, Mach revealed that he had come to understand something about the foundations of Newtonian mechanics that neither he, nor very many others, had understood before. And this improved understanding clearly arose from his study of the literature on inertial systems" (DiSalle, 2002a, p.175). DiSalle goes on to argue that Mach had learned to distinguish between (1) the "external question" about whether the law of inertia could be replaced with an alternative formulation, and (2) the "internal question" of how we must go about constructing inertial reference frames. These two questions essentially correspond to the distinction mentioned earlier between (1) the *causal* question of what physical phenomena may be responsible for inertial motion, and (2) the *epistemological* question of how we may derive knowledge of an inertial system. As we saw, these two questions were conflated in Neumann's approach, whereas Lange did not conflate them but only attempted to answer the first question. The fact that Mach shows his great admiration for the work of Ludwig Lange demonstrates that he was not merely concerned with the causal question, but that both of these questions troubled him. It is highly likely that in his early work, Mach did not distinguish precisely between the causal and epistemological problems, since after all he did not notice how these two problems were conflated in Neumann's work. Moreover, Mach's unwillingness to use explicitly causal language certainly did not help to shed light on the distinction.

6.1. DiSalle's criticism of Einstein

In defending Mach however, DiSalle deflects the criticism that Stein and others had directed at Mach towards "Mach's 20^{th} Century acolytes", notably Einstein. DiSalle claims that Einstein and Reichenbach's ideas "frequently involve confusion about the nature of the principle of inertia." Einstein argued that inertial frames in Newton's theory function as a "factitious cause" of inertial effects in Newtonian mechanics and special relativity; he attributed this idea to Ernst Mach and illustrated it using a thought experiment in (Einstein et al., 1952, p.112-113). This thought experiment can be paraphrased as follows: we consider two bodies fixed in space, S_1 and S_2, which are in relative rotation with respect to one another around the line which connects each to the other. S_1 is found to be perfectly spherical while S_2 bulges at the equator. Here Einstein asks, what is the reason for the difference between the two? According to Newtonian Mechanics, Einstein claims: "The laws of mechanics apply to the space R_1, in respect to which the body S_1 is at rest, but not to the space R_2, in respect to which the body S_2 is at rest. But the privileged space R_1 of Galileo, thus introduced, is a merely factitious cause, and not a thing that can be observed." Einstein then offers the Machian explanation:

> The only satisfactory answer must be that the physical system consisting of S_1 and S_2 reveals within itself no imaginable cause to which the differing behaviour of S_1 and S_2 can be referred. The cause must therefore lie outside this system. We have to take it that the general laws of motion, which in particular determine the shapes of S_1 and S_2, must be such that the mechanical behaviour of S_1 and S_2 is partly conditioned, in quite essential respects, by distant masses which we have not included in the system under consideration.

In his essay, DiSalle claims not only that Einstein commits a "philosophical mistake" in his reasoning here, but also that these ideas are in fact in contradiction with Mach's own thoughts. We will examine both these claims in what follows.

DiSalle's rebuttal of Einstein is founded on what he sees as a comparatively subtle understanding of inertial frames which he explicates as follows: "an inertial system is not itself a cause, but constitutes the framework within which causal efficacy is measured, through the accelerations that causal agents produce in one another." Furthermore, DiSalle argues that this is the view that Mach came to understand in his later writings based on his discussions of Newton's fifth corollary. According to DiSalle, Mach was not granting causal status to the fixed stars, rather he was just claiming that "in actual practice, the stars constitute the *empirical* framework within which causal influences, at least among the celestial bodies, are measured." Concerning Mach's speculative hypothesis about inertia coming from some action of the fixed stars, DiSalle (2002a) writes:

> Of course Mach's speculation about the origin of inertia suggests that the stars may be playing a causal role as well, but Mach clearly understood this as a separate issue—a question about a possible alternative theory, rather than the identification of an internal "epistemological defect" of the theory of inertial systems.

Einstein, on the other hand, had confused these two issues together, claiming that the Machian hypothesis about the fixed stars implied an "epistemological defect" in classical mechanics.

6.2. Defence of Einstein

Now, while we can grant to DiSalle that perhaps Einstein did not express himself entirely accurately, it would be a stretch to claim that his ideas here are not reflective of Mach's own writings. What we will argue is that Einstein's argument takes as its premise one of Mach's hypotheses, but that given this premise, Einstein is correct in viewing the question of inertial reference frames as an epistemological defect in classical mechanics. To clarify the situation, we will need to return to our discussion of Mach's views concerning causality.

At the heart of Mach's critique of Newton, as we have argued in earlier parts of this paper, is an attempt to overturn the prevailing view of causation according to which singular causes, represented by forces, are responsible for singular changes in motion. This view is epitomised by the definitions of *force* provided by Poisson and Maxwell cited previously. Accordingly, the classical law of inertia is a necessity since it defines the standard for how motion proceeds in the absence of external causes effecting it. Now it is important to note that DiSalle's defence of Newtonian inertial systems, on the basis of which he claims that Einstein commits a "philosophical mistake", is predicated upon this very conception of causality (DiSalle, 2002a, p.181):

> an inertial system is not itself a cause, but constitutes the framework within which causal efficacy is measured, through the accelerations that causal agents produce in one another.

But we have seen that, at least in his early writings, Mach contradicts this view. Mach's primary innovation, we have argued, is in fact to raise the possibility that inertial motion is not *uncaused* motion, but rather could be a motion causally determined by the regularity of the cosmic environment constituted of the distant stars. According to *this* view, which is one of the two hypotheses which Mach proposes as alternatives to that of Newton, inertial frames would only need to make an appearance in our physics when analysing subsystems of the universe. This is because, when the universe as a whole is taken into consideration, it would become totally arbitrary whether we use a coordinate system in which the bulk of matter is stationary or rotating around a common point. Our inertial system, which would be necessary to consider in a subsystem of the universe, would thus be acting as an abbreviated reference to the rest of the matter in the universe; and it would indeed, in this case, be *causally* responsible for determining inertial motion in that subsystem. Einstein's argument therefore is perfectly Machian. Furthermore, it is not based on a misunderstanding of inertial systems, rather, Einstein is merely taking for granted Mach's second hypothesis (Hypothesis 4.2) and the expanded Machian view of causality implied in it. According to this view, inertial systems *do* appear as the "factitious cause" of inertial effects, therefore there *is* an "epistemological defect" in the theory if we cannot identify some observable structure as causally responsible for these effects.

But why then, we might ask, did Mach seem to go against this view in his later years? It is not that Mach abandoned the views of his younger years, but rather, as DiSalle recognises, that he came to appreciate Lange's solution to the epistemological problem concerning the definition of Newton's first law, and even recognised the possibility of interpreting Newton's fifth corollary in a similar way. Mach had found, in Lange's solution, the true empirical content of Newton's law, which he could now recognise as a genuine "*hypothesis*". Stripped of its metaphysical reference to absolute space, Newton's law now became a hypothesis among others that could be subjected to empirical scrutiny by future experiments. The possibility of alternative hypotheses, involving different conceptions of the causal relations of things, such as the hypotheses Mach explored in his early writings, is not ruled out by Mach.

6.3. Einstein's further hypotheses

In addition to what we might call the *classical inertial hypothesis*: that in the absence of identifiable forces, all bodies move rectilinearly and uniformly with respect to one-another, two other *implicit* hypotheses accompany classical mechanics. In his formerly quoted essay, Einstein identifies these implicit hypotheses and expresses them as follows (Einstein et al., 1952, p.112):

(1) "To two selected material points of a stationary rigid body there always corresponds a distance of quite definite length, which is independent of the locality and orientation of the body, and is also independent of the time."

(2) "To two selected positions of the hands of a clock at rest relatively to the privileged system of reference there always corresponds an interval of time of a definite length, which is independent of place and time."

The first is connected to the Euclidean nature of absolute space, while the second depends on the homogeneity and universality of absolute time. Newton's mechanics can be interpreted in two ways: either we could consider that these laws are the laws of geometry, they belong to the space-time structure of Newtonian theory and need no causal explanation. Or else, we might postulate that some '*medium*' or '*metrical field*' is causally responsible for determining the size of rigid bodies and the relative speed of different clocks. The first interpretation corresponds to the view DiSalle defends, while the second opens itself to Machian criticism.[26] Einstein asserts that in his theory of general relativity, Newton's hypotheses are overturned; indeed in Einstein's theory these laws are deemed to be only approximate and contingent truths that will hold if the matter distribution in the cosmos is entirely homogeneous and certain further conditions about cosmic structure are satisfied. In other words, Newton's hypothesis was wrong, and since it was wrong, is it not reasonable to consider whether the error may stem from the classical conception of causality which Newton and others in his time assumed? It is particularly revealing that while DiSalle characterises Einstein's Machian reasoning as a "philosophical mistake", he admits that "it has to be considered a fortunate one for the history of physics." For *what world do we live in after all?* A Newtonian world, in which there is no reason to question whether our inertial reference frames might be conditioned by the action of other matter? Or do we live in a world in which such conditioning does take place? If Einstein's Machian intuitions were, at least to some extent, correct, does it matter that his reasoning contradicts the mathematical formalism of inertial systems?

[26] Concerning the relational definition of a preferred temporal metric, Mittelstaedt (1980), Barbour (1981) have explored this idea and called it the 'second Mach's principle' (Although Mach himself did not explicitly tackle this issue). Barbour and Mittelstaedt do not consider similar arguments for the relational definition of scale, although this has more recently become an important part of Barbour's work (Barbour, 2010, 2023).

7. Closing comments

At the core of Mach's principle is a challenge to the classical conception of inertia as uncaused motion. In the standard approach to Newtonian mechanics, a force acts as a cause, producing a change in motion. By necessity, therefore, a law of inertia is conceived which defines motion in the absence of causes. Mach insists however, that all motion is equally caused by the surrounding circumstances. We are not justified in our treatment of some effect to single out some particular circumstance as the cause but must take heed of the influence of the entire universe. We must interpret the homogeneity of inertial motion as an effect of some homogeneous aspect of the environment with respect to which this motion is performed. Although this revision of the view of causality that we here claim was operative in Mach's critique of inertia was not precisely articulated in Mach's treatment of the issue, in part due to Mach's deliberate resistance to using causal language, we saw that Mach does explicitly discuss this revision of causality in his early work, at a time which coincides with his original speculations on the origin of inertia.

Second, we distinguished Mach's principle, which is a purely *formal* affirmation of relationalism, from Mach's hypotheses, of which we identified two classes. The second class of hypotheses is the one proper to Mach, which we may thereby call the *Mach hypothesis* (Hypothesis 4.2). According to this hypothesis, Mach proposes that the distant masses of the universe may be responsible for the causal determination of so called "inertial" motion. This intriguing thought has been the subject of much speculation ever since. In his early years, Einstein took it quite seriously, and used it as a guiding principle during the development of his theory of general relativity. While DiSalle questions the motivation behind Einstein's reasoning, defending the mathematical formalism of reference frames as an adequate account of the phenomenon of inertia, and argues that Mach came to acknowledge this view, we saw that this conception contradicts Mach's original intention to regard inertia as a form of *caused* motion.

Going forwards, there is no doubt that the insights drawn from this study will help to clarify the basis and motivation underlying speculative Machian theories. Moreover, the distinction drawn between Mach's principle and the Mach hypotheses may shed some light on the controversial question of the Machian status of general relativity.

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