On Sciama 1953

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Abstract

This is a shortened version of a draft I shared with Alexander Unzicker which he has spoken about in a recent YouTube video that has received some attention (link on my website). In this article, I discuss Dennis Sciama's simple Machian model of inertia and gravitation and draw out its philosophical implications. The article was originally written in January 2023.

Contents

1 Vector Model of Inertial Forces

We begin our investigations by inquiring as to how we can extend the relativity of position and velocity in Newtonian mechanics to a broader relativity principle encompassing accelerations and uniform rotations of the reference system, in short, the *rigid transformations*; which are the transformations that preserve the spatial relations between points but change arbitrarily the relation of a given point to space. The motivation for this is the assertion that our concept of space (for Newton it is called 'absolute space') is not something physical, instead it has a merely *methodological* role, so we cannot appeal to it to pick out a preferred class of reference frames.

Quite apart from Mach, the mathematician Carl Neumann pointed out that the Newtonian-Galilean law of inertia, which states that *in the absence of any force bodies move in rectilinear and uniform motion* suffers from an inadequate definition. It is not stated with respect to what object rectilinear and uniform motion is to be defined. Clearly we cannot use the earth or the sun, although these work as approximations for a limited set of phenomena. Neumann proposed to solve this problem by considering a hypothetical 'Body Alpha' with respect to which rectilinearity would be defined [Neumann, 1993]:

> '[A] material point left to itself proceeds in a straight line — i.e., in a path that is rectilinear in relation to this Body Alpha.'

Mach on the other hand famously suggested to solve the problem by deriving the inertial frame from a suitably defined mean of all matter in the universe. The hypothesis that the inertial law should be modified in this way is sometimes refered to as '*Mach's principle*', however we will call it the *Mach Hypothesis* to emphasize its hypothetical character. It is worth noting that Mach does not provide a decisive replacement for the inertial law in his own work [Mach, 1893, Mach, 1872].

Later in this piece we will investigate the work of Dennis Sciama [Sciama, 1953a], who took up Mach's challenge and developed an inherently cosmological vector potential theory of inertia in which the gravitational force is derived as a side-effect of the inertial law in a relational cosmos. Subsequently, we will also see how this theory might be extended to provide a relational account of metrical propeties of bodies as well as their inertial properties (this is the ultimate aim of this project). For now however, we put aside Mach's hypothesis and focus on generalising Newtonian inertia to non-inertial frames without considering the coupling of the inertial field to matter.

1.1 Non-relativistic Derivation of Inertial Forces

This subsection is not contributing anything new mathematically, it is simple a derivation of the centrifugal and Coriolis forces in classical mechanics.

It is clear that if we wish to extend Newton's law of inertia to non-inertial frames, inertial forces will make an appearance. For simplicity we will consider a Newtonian system with zero gravity. In an *inertial frame* S', particles follow rectilinear paths of constant velocity thought absolute space. By the *principle of relative motion*, it is necessary that the laws of physics do not change if we transfer to a reference frame S which is in uniform rotation w.r.t. the original reference frame.[1] However the paths in this new frame are no longer rectilinear and uniform. Therefore it will be necessary to consider a force field along with the bodies that is responsible for the appearance of inertial forces when we transfer to S.[2]

An arbitrary vector in S is parameterised in a Cartesian basis as follows:

$$\bar{x} = x_1\hat{x_1} + x_2\hat{x_2} + x_3\hat{x_3} \tag{1}$$

It's derivative w.r.t. time in S is given by:

$$\frac{d\bar{x}}{dt} = \frac{d}{dt}\left(x_1\hat{x_1} + x_2\hat{x_2} + x_3\hat{x_3}\right) \tag{2}$$

$$= \left(\frac{dx_1}{dt}\hat{x}_1 + \frac{dx_2}{dt}\hat{x}_2 + \frac{dx_3}{dt}\hat{x}_3\right) + \left(x_1\frac{d\hat{x}_1}{dt} + x_2\frac{d\hat{x}_2}{dt} + x_3\frac{d\hat{x}_3}{dt}\right) \tag{3}$$

Now in the rotating (coordinate) frame S, the original coordinate basis of S' moves according to:

$$\frac{d\hat{x}_i}{dt} = -\bar{\omega} \times \hat{x}_i \tag{4}$$

where ω is the angular velocity of the rotation of S' w.r.t. S. For simplicity, we have chosen that $\bar{\omega}$ represents a rotation around the origin so that x is a radial vector: $\bar{x} = \bar{r}$.

Therefore, if we interpret r as a position vector in a given coordinate system, then $\frac{d\bar{r}}{dt} = \bar{v}$ is the velocity in that coordinate system, then the velocity in a non-rotation coordinate system S' is given in terms of the velocity in the rotating system S and ω as:

$$\bar{v}|_{S'} = \bar{v}|_S + \omega \times \bar{x} \tag{5}$$

[1] We will be using this notation throughout this piece, where S' corresponds to the inertial or later 'physical' reference frame, while S corresponds to our arbitrary chosen coordinate system.

[2] Informal remark: There may be some similarity between what is being done here and Gryb and Gomes' '*Turbocharging relationalism*' paper [Gomes and Gryb, 2021], in which inertial effects are attributed to some absolute charge. However, the present approach is meant to be very technically simple.

where $|_{S'}$ means evaluated in the inertial coordinate system, and $|_S$ means evaluated in the rotating coordinate system.

We can apply the same procedure to velocity vectors to derive an expression for acceleration under a change of coordinates:

$$\bar{a}|_{S'} = \frac{d^2\bar{r}}{dt^2}\bigg|_{S'} = \left(\frac{d}{dt}\bigg|_S + \bar{\omega}\times\right)^2 \bar{r} \tag{6}$$

$$= \frac{d^2\bar{r}}{dt^2}\bigg|_S + \frac{d}{dt}(\bar{\omega}\times\bar{r})\bigg|_S + \bar{\omega}\times\frac{d\bar{r}}{dt}\bigg|_S + \bar{\omega}\times(\bar{\omega}\times\bar{r}) \tag{7}$$

Assuming the rotation of S is uniform, then $\frac{d}{dt}\bar{\omega} = 0$, therefore the expression reduces to:

$$\frac{d^2\bar{r}}{dt^2}\bigg|_{S'} = \frac{d^2\bar{r}}{dt^2}\bigg|_S + 2\bar{\omega}\times\frac{d\bar{r}}{dt}\bigg|_S + \bar{\omega}\times(\bar{\omega}\times\bar{r}) \tag{8}$$

Now since S' is an inertial frame, there is no acceleration: $\frac{d^2\bar{r}}{dt^2}\Big|_{S'} = 0$, therefore we get an expression for the acceleration in S:

$$\bar{a}|_S = -2\bar{\omega}\times\bar{v}|_S - \bar{\omega}\times(\bar{\omega}\times\bar{r}) \tag{9}$$

$$= -2\bar{\omega}\times\bar{v}|_{S'} + \omega^2\bar{r} \tag{10}$$

given that ω is orthogonal to r.

As such, we have derived the centrifugal ($\omega^2 r$) and Coriolis ($2\bar{\omega}\times\bar{v}$) accelerations for a rotating frame of reference. If we want to be able to generalise the laws of physics to non-inertial frames of reference therefore, it is necessary that an *inertial field* exist in space, which is responsible for producing these forces when it is moving in our frame of reference.

1.2 Relationship between Inertia and Electromagnetism

It is well known that it is possible to describe the centrifugal and Coriolis forces in terms of scalar and vector potentials respectively. These are given by:

$$\Phi = -\frac{1}{2}(\bar{\omega}\times\bar{r})^2 \tag{11}$$

$$\bar{A} = \bar{\omega}\times\bar{r} \tag{12}$$

where Φ is the centrifugal potential and $\bar{A}$ is the Coriolis potential. This way, we can recover the centrifugal ($\omega^2 r$) and Coriolis ($2\bar{\omega}$) fields in analogy with electromagnetism from the following:

$$\begin{aligned}
\bar{E}_{cent} &= \frac{\partial}{\partial t}\bar{A} - \nabla\cdot\Phi \\
&= \nabla\cdot\left(\frac{1}{2}(\bar{\omega}\times\bar{r})^2\right) + \frac{\partial}{\partial t}(\bar{\omega}\times\bar{r}) \\
&= \omega^2\bar{r} \\
\bar{B}_{Cor} &= \nabla\times\bar{A} \\
&= \nabla\times(\bar{\omega}\times\bar{r}) \\
&= -2\bar{\omega}
\end{aligned} \tag{13}$$

Since we have considered uniform rotation, the second part of the Centrifugal force, $(\frac{\partial}{\partial t}(\bar{\omega}\times\bar{r}))$ is zero.

It is easy to see, that the above equations are formally analogous to the equations governing electromagnetism; where the centrifugal force is analogous to the electric and the Coriolis force is analogous to the magnetic.

The main difference between this and actual electromagnetism is that while the electromagnetic vector and scalar potentials are given by integrals over the charge and current densities in nearby matter, the inertial potentials, so far considered are not given by a relation to matter, but rather, by a relation to an invariant inertial field.

In later sections of this work (see section 2), we will see that if the analogy with electromagnetism is carried further, and we define the inertial potentials in terms of an integral over matter, this (1) leads directly to a gravitational theory which was proposed by [Sciama, 1953a, Sciama, 1953b], furthermore (2) it fulfills Mach's hypothesis, (3) predicts the existence of gravity, (4) anticipates the equivalence principle and (5) gives an explanation of the size of the Newtonian constant G. But we will get to all this in due course. In this section we are not yet considering that the inertial potentials are coupled to matter; rather, we can consider these as absolute, they are simply properties of some field that Neumann might have called the 'Body Alpha', but we may as well call the *inertial field*.

2 Sciama's Vector Model of Gravity and Inertia

Instead of starting from a special-relativistic description of inertial forces, Dennis Sciama's 1953 model given in a short paper titled '*On the Origin of Inertia*' [Sciama, 1953a] begins with the more basic classical account and its analogy to electromagnetism that we discussed in section 1.

In his paper, gravelectric and gravo-magnetic (i.e. inertial) forces are given in terms of a scalar potential $\Phi = \frac{1}{2}(\bar{\omega} \times \bar{r})^2$ and vector potential $\bar{A} = \bar{\omega} \times \bar{r}$. We can observe that the Coriolis potential $\bar{A}$ corresponds to the velocity $\bar{u} = \bar{\omega} \times \bar{r}$ of the inertial frame S' in S. Now if we consider an extension of $\bar{A} = \bar{u}$ to a four velocity: $A_\mu = u_\mu$, we will require that the magnitude of the four-velocity vector is always c. from this we can deduce the A_0 component:

$$c^2 = A_0^2 - A_1^2 - A_2^2 - A_3^2 \tag{14}$$

$$= A_0^2 - \bar{u}^2 \tag{15}$$

$$A_0^2 = c^2(1 + \bar{u}^2/c^2) \tag{16}$$

$$A_0 = -c\,(1 + \bar{u}^2/c^2)^{1/2} \tag{17}$$

Now for small values of $\bar{u}$, we can approximate A_0 to first order using a Taylor expansion:

$$\frac{A_0}{c} \approx -\left(1 + \frac{1}{2}\frac{\bar{u}^2}{c^2} + ...\right) \tag{18}$$

Now we can see that this scalar part of the four-vector potential corresponding to the four-velocity of the 'inertial field' in our coordinate frame is proportional to the scalar potential $\Phi = -\frac{1}{2}\bar{u}^2$ (ignoring the normalisation by c) which was derived as the source of the centrifugal force. Since the centrifugal field depends only on the divergence of Φ the added constant -1 has no dynamical effect. Therefore it is reasonable to interpret the scalar centrifugal potential as the zeroth component of the four-velocity A_μ of the inertial frame.

In this way, the centrifugal and Coriolis forces are unified by this special relativistic trick; i.e. by interpreting the centrifugal potential Φ as the modification of the zeroth component of a relativistic four-vector generalisation of the Coriolis potential $\bar{A}$. It is worthwhile mentioning, however, that Sciama's model is purely heuristic and not fully consistent with special relativity.

2.1 Source of the Potential

Now that we can describe inertial forces in terms of four-vector potential A_μ in analogy with electromagnetism, the most natural thing to do is to continue this analogy by writing A_μ as an integral over the relevant material sources. This is exactly what Sciama did, and amazingly the result leads directly to a fully Machian theory of gravity and inertia since it enables a fully relational definition of inertia and at the same time predicts the existence of gravity!

While for electromagnetism the material source is the charge and current distribution of nearby matter, in our case the source of the inertial potential is going to be the four-momentum distribution of matter throughout the cosmos. Strictly speaking, in electromagnetism, the potential ought also to be an integral over the volume of the whole observable universe, however, since there are both positive and negative charges, distant matter which is on average neutral becomes irrelevant. This is no longer the case for our relationally defined inertia; since there are only positive charges, nothing cancels anything else out, and so our four-potential will be dominated by the influence of very distant matter. Still in analogy with electromagnetism, we write the four-vector potential as the integral:

$$A_\mu(x) = -\int_V \frac{p_\mu(x')}{c\,r} dV \tag{19}$$

where $p_\mu = (\rho\sqrt{c^2 + \bar{v}^2},\, \rho\, v_1,\, \rho\, v_2,\, \rho\, v_3)$, ρ is the mass-density, v_i are the components of the velocity of said mass. $A_\mu(x)$ is the potential at a given point x, while the spatial coordinate represented by x' is the past light-cone seen from x (i.e. the observable universe seen from x). The distance r is the distance between x and x' in these coordinates. The volume dV we are integrating over is the volume of the past light-cone with coordinates x'. Naturally, this potential depends on the frame of reference considered, as the velocities of the masses in the universe will vary according to the frame we choose.

From this potential, we can carry on the procedure in analogy with electromagnetism to derive 'gravelectric' and 'gravomagnetic' fields. Since the 'gravomagnetic' (or Coriolis) field $\bar{B}$ has no influence on a test particle in its own rest frame as $\bar{B}$ only becomes relevant for bodies moving in that frame, we only need to consider the gravelectric field $\bar{E}$ to determine inertial motion. Inertial motion is defined by choosing a frame for the particle in which the gravelectric force will be zero, since this will be its rest frame. As Sciama explains [Sciama, 1953a, p.35].:

> In Newtonian language we could say that the universe moves relative to any body in such a way that the body never experiences a force—the difference from ordinary Newtonian theory being that the forces acting on a body are derived entirely from the matter of the universe.

Like in electromagnetism, the components of the gravelectric field $\bar{E}$ and gravomagnetic field $\bar{B}$ are given by:

$$\bar{E} = -\frac{1}{c}\partial_t \bar{A} - \bar{\nabla} A_0 \qquad\qquad \bar{B} = \bar{\nabla} \times \bar{A} \tag{20}$$

2.2 A Homogeneous Universe

At first, Sciama considers a homogeneous and isotropic universe of uniform density $\rho(x) = \rho$, expanding according to the Hubble law: $\bar{v} = \bar{r}/\tau$. The zeroth component of the potential is given by:

$$A_0 = -\int_V \frac{\rho}{\bar{r}} dV \tag{21}$$
$$= -\int_{r=0}^{c\tau} 4\pi r^2 \frac{\rho}{r} dr \tag{22}$$
$$= -4\pi\rho \int_{r=0}^{c\tau} \frac{1}{r} dr \tag{23}$$
$$= -2\pi\rho c^2\tau^2 \tag{24}$$
$$= \Phi \tag{25}$$

Where we define $\Phi = -2\pi\rho c^2\tau^2$ for use later on. Due to symmetry, all other components of A_μ vanish: $A_1 = A_2 = A_3 = 0$. This doesnt tell us very much at all since the spatial derivatives of A_0 will trivially be zero, therefore $\bar{E}$ will also be zero.

However, if we consider that the universe as a whole is moving at non-zero velocity $\bar{v}_m = (v_1,\, 0,\, 0)$ w.r.t. the test particle, then we will have:

$$A_1 = -4\pi\rho \int_{r=0}^{c\tau} r \left(\frac{\bar{v_1}(r)}{c\tau} - \frac{v}{c} \right) dr \tag{26}$$
$$= v_1 \Phi \tag{27}$$

while the other components are again: $A_2 = A_3 = 0$. Inertial motion, for this test particle will therefore be defined by:[3]

$$\bar{E} = E_1 = \partial_1 A_0 - \partial_0 A_1 = 0 \tag{28}$$

Now since $\partial_1 A_0 = 0$, we simply have:

$$\partial_0 A_1 = 2\pi\rho c\tau^2 \frac{\partial v_1}{\partial t} = 0 \tag{29}$$

Therefore the equation of motion is simply: $\frac{\partial v_1}{\partial t} = 0$ as expected. Naturally, the same procedure can be applied to derive $\frac{\partial v_2}{\partial t} = \frac{\partial v_3}{\partial t} = 0$. In contrast to the derivation of the inertial law in section 1, in this case it is defined in an entirely relational way without any appeal to absolute space or any kind of physical structure which exists over and above the masses of the universe. In this way, Sciama's theory fulfills Mach's hypothesis since the inertial potential has no autonomous existence and expresses only a relation to matter.

To be sure however we should examine whether the correct inertial forces are also recovered in a rotating frame of reference.

2.3 A Rotating Homogeneous Universe

So far we have seen how Sciama derives the classical law of inertia in an inertial reference frame $\frac{\partial v_1}{\partial t} = 0$ in a purely relational cosmos. Now if this description is to fulfill Mach's hypothesis, it must similarly derive inertial forces in a rotating frame of reference; i.e. the *centrifugal* and *Coriolis* forces.

We consider a similarly homogeneous, but now *rotating* universe, with angular velocity $\bar{\omega} = \omega\hat{x}_3$, rotating around the x_3 axis. By defining Φ such that if $\omega = 0$ we have $A^0 = \Phi$, i.e.

[3]Here I am using ∂_0 interchangeably with $\partial_t = \frac{\partial}{\partial t}$, and $\partial_1 = \frac{\partial}{\partial x_1}$.

$\Phi = -2\pi\rho c^2\tau^2$, we can calculate the components of A^μ when ω is non-zero:

$$A^1 = -\frac{\Phi}{c}\omega x_2 \tag{30}$$
$$A^2 = \frac{\Phi}{c}\omega x_1 \tag{31}$$
$$A^3 = 0 \tag{32}$$
$$A^0 = \Phi(1 + \omega^2 r^2/c^2)^{1/2} \tag{33}$$

The transformation of the A^0 component follows from the transformation properties of four-vectors in a rotating coordinate system. Sciama borrowed this from a paper by Nathan Rosen [Rosen, 1947]. The reason for this was already covered at the start of section 2 (see the "special relativistic trick" mentioned there).

Gravelectric field: Now since A^0 is no longer independent of the spatial coordinate, we will have induced a centrifugal force:

$$\bar{E} = \bar{\nabla} A^0 - \frac{1}{c}\frac{\partial \bar{A}}{\partial t} \tag{34}$$
$$= \Phi \frac{\omega^2 \bar{r}/c^2}{(1 + \omega^2 r^2/c^2)^{1/2}} \tag{35}$$

where $\bar{A} = (A^1, A^2, A^3)$. For particles near the origin where $r > 0$ we have $\bar{E} \approx \Phi\omega^2\bar{r}/c^2$ which is the centrifugal force. For the purposes of his model, which is not a rigours theory, Sciama simply ignores the factor of $(1 + \omega^2 r^2/c^2)^{-1/2}$. It is not obvious to tell how to deal with this in a fully relativistic manner; I have tried to crack it myself without much success. It appears that in order to treat relativistic inertial forces in analogy with electromagnetism rigorously, a much more involved process is required, since we need to consider with respect to which coordinates exactly our forces are defined. This question is covered in some depth by [Rizzi and Ruggiero, 2004], but for our purposes we will not go into this further.

As we will see, the metrical theory I develop in section **??** avoids these difficulties since the accelerations considered are always clearly referred to the coordinate frame.

Gravomagnetic field: Due to the rotation of the universe we also have a non-zero gravomagnetic field:

$$\bar{B} = \bar{\nabla} \times \bar{A} \tag{36}$$
$$= (\partial_2 A_1 - \partial_1 A_2)\hat{x}_3 \tag{37}$$
$$= -2\frac{\Phi}{c}\omega\hat{x}_3 \tag{38}$$

For particles moving at a velocity v in this reference frame we get an induced gravelectric field:

$$\frac{1}{c}\bar{v} \times \bar{B} = -2\,\frac{\Phi}{c^2}\,\bar{v} \times \bar{\omega} \tag{39}$$

This field is produces the Coriolis acceleration, and like the centrifugal acceleration, it is attributed to the action of the masses of the universe. And so Sciama writes:

> "Thus in our theory we can regard the earth as stationary and the Foucault pendulum as pulled around by the gravomagnetic field of the rotating universe." [Sciama, 1953a, p.41]

Example of inertial forces at work: For a simple example of the centrifugal and Coriolis forces in action, we may consider a particle in our rotating universe at a small distance $\bar{r} = (r, 0, 0)$ from the origin. The particle is moving along with the rest of the rotating universe so that it is stationary in relation to the masses of the universe, but in our reference frame it is moving. Therefore in our frame of reference the particle has velocity: $\bar{v} = (0, r\omega, 0)$. Now the combination of the centrifugal[4] and Coriolis forces will cause an acceleration of the particle in this frame given by:

$$\bar{a} = G\left(\frac{\Phi}{c^2}\omega^2\bar{r} - 2\frac{\Phi}{c^2}(r\omega)\omega\hat{x}_1\right) \tag{40}$$

$$= G\left(\frac{\Phi}{c^2}\omega^2\bar{r} - 2\frac{\Phi}{c^2}r\omega^2\hat{r}\right) \tag{41}$$

$$= -\frac{G\Phi}{c^2}\omega^2\bar{r} \tag{42}$$

where G is a normalisation constant which converts the gravelectric field into an acceleration. In order to make things consistent with the same situation in a non-rotating frame, we require simply that $G = c^2/\Phi$, and recover the equation for circular motion of the particle around the origin:

$$\bar{a} = -\omega^2\bar{r} \tag{43}$$

In this case we can interpret the situation as a case of a rotating universe inducing the centripetal acceleration in particles co-moving with it. The sum of the induced centrifugal force (pointing outwards) with the induced Coriolis force (pointing inwards) produces a net centripetal force towards the origin, keeping the body in circular motion, and therefore stationary w.r.t. the rotating universe. As we will soon see, the constant G which was introduced to convert $\bar{E}$ into an acceleration will turn out to be the Newtonian gravitational constant. Its relationship to the potential of the universe $G = c^2/\Phi$ is no accident since, as we shall see, according to this Machin theory, the strength of the gravitational force is inversely related to the sum of the rest of the masses in the universe.

2.4 An Inhomogeneous Universe: Gravity

If we consider an irrotational homogeneous universe of density ρ, but now add an additional mass M at the origin, the inertial field will no longer be homogeneous. For a given test mass at a distance r from the mass M, A_0 will now be:

$$A_0 = \Phi + \phi \tag{44}$$

where $\phi = -M/r$ is the potential of the nearby mass M. Now if the universe + central mass M is moving at non-zero velocity $\bar{v}$ w.r.t. the test particle's rest frame, then we will have the vector potential:

$$\bar{A} = (\Phi + \phi)\frac{\bar{v}}{c} \tag{45}$$

Now our condition for inertial motion is given by $\bar{E} = 0$. The Gravelectric field is given by:

$$\bar{E} = \bar{\nabla}A_0 - \frac{1}{c}\frac{\partial\bar{A}}{\partial t} \tag{46}$$

$$= -\frac{M}{r^2}\hat{r} - \frac{(\Phi + \phi)}{c^2}\frac{\partial v}{\partial t} \tag{47}$$

[4] Ignoring the relativistic factor of $(1 + \omega^2 r^2/c^2)^{-1/2}$.

Therefore inertial motion of the particle w.r.t. the universe will take the form:

$$\frac{\partial v}{\partial t} = -\frac{c^2}{(\Phi + \phi)} \frac{M}{r^2} \hat{r} \tag{48}$$

Now since $\Phi >> \phi$ this term at the front is approximately: $\frac{c^2}{(\Phi+\phi)} \approx \frac{c^2}{\Phi}$ which we have previously abbreviated as the normalisation constant G. Imputing it here, we recover Newton's law of universal gravitation:

$$\frac{\partial v}{\partial t} = -G\frac{M}{r^2}\hat{r} \tag{49}$$

And we recognise G as the Newtonian gravitational constant. But the difference is now that G is not regarded as a universal constant, rather, it is inversely proportional to the gravitational potential of the entire universe. The smallness of G and thus the weakness of the gravitational force is simply an outcome of the vastness of our observable universe in relation to any nearby mass. Moreover, the apparent constancy of G is merely a consequence of the relative constancy of the cosmological environment.

The most groundbreaking result of Sciama's model however, is that gravity *necessarily* exists as a direct result of Mach's principle, that is, of the necessity to define a law of inertia in a wholly relational manner, in the absence of any absolute physical structures. Since the mass of the universe is not homogeneously distributed, the inertial field which defines motion in the absence of other forces, cannot be homogeneous either. The expression of this necessary inhomogeneity of the inertial field is simply what 'gravity' is. This is the key feature that distinguishes Sciama's model from other gravitational theories such as general relativity theory. Gravity arises in this model *necessarily*, it is not something we have to assume before hand and fit to known data. There is no 'Newtonian fit'.[5]

For this reason, we can regard Sciama's theory as explaining the existence, indeed the necessary existence of gravity. This result, along with the natural explanation of the equivalence principle (the unity of gravity and inertia) is perhaps the strongest piece of evidence in favour of Mach's hypothesis.

If we reject Mach's hypothesis, we still have to explain why gravity exists at all. *Why should the inertial field couple to matter? Why don't we live in a flat Minkowski space? Why isn't G just zero, why does it take the particular value it has?* These are all questions which in my view general relativity fails to answer, yet all of these have a very reasonable, logical explanation according to the model described above.

2.5 Problem with the model

- Sciama's model is not fully consistent with special relativity.
- No account is given of the metrical effects of gravity, i.e. of the fact that gravity effects how material blocks stack and how material clocks run.
- While the model recognises frames related to one another by rigid transformations as physically equivalent, it does not account for symmetry under universal scale transformations.[6]

Sciama own attempt to develop a relativistic extension of the model which could describe the behaviour of photons, involved giving up on the vector-potential approach. In his PhD thesis, Sciama argued that his vector potential theory was incomplete because:

[5] It should be noted that the first person historically to suggest that gravity would arise *necessarily* as a consequence of the relativity of inertia is in fact Hans Reissner [Reissner, 1915], although I was not aware of Reissner's paper when I wrote the above paragraph. Reissner, as well as Schödinger in 1925 [Schrödinger, 1995] both derive similar expressions for the gravitational constant G, however they do not use an analogy with electromagnetism.

[6] This is a question I will address personally in future work.

> "in a theory based on a vector potential it is difficult to give a consistent relativistic discussion of the structure of the universe as a whole. In addition it is difficult to describe the motion of light in a gravitational field." [Sciama, 1953b, p.55]

A little later on [Sciama, 1953b, p.56-58] Sciama derives a tensorial expression for inertial forces, and explains that the gravitational potential "cannot be a vector but must be a tensor of the second rank." It is not clear to me that Sciama was right about this, I would not personally abandon the attempt to model gravity using a vector potential.

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