On the Possibility of Deriving Gravity as the Direct Consequence of the Relativity of Inertia

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Abstract

[Translator's note]: This is an English translation of a paper by Hans Reissner originally published in 1915. Hans Reissner was an accomplished aeronautical engineer and theoretical physicist; the first person in history to design a functional fully metal aircraft (the *Reissner canard*), and also the first to derive the *Reissner-Nordström metric* for a spherically symmetric, charged, stationary mass in general relativity. This paper, which is a follow-up from a pevious paper Reissner published in 1914, offers insight into a strategy for implementing Mach's principle, however it goes further than what Mach suggested, since Reissner shows that according to his theory, we can interpret gravity as arising *necessarily* as a consequence of the relativisation of inertia. In other words, Reissner offers a hypothetical explanation for the existence of gravity. Historically, Reissner appears to be the first person to have explicitly raised this possibility. Although the paper was formerly partially translated by Julian Barbour, I am providing a full translation here to help highlight the unique aspects of this second paper that were lacking in the first.

Classical mechanics introduces inertia and gravity as independent phenomena and sees inertia as a resistance to acceleration in relation to absolute space. The fact that both of these forces are proportional to the same mass appears in classical mechanics as a coincidental relationship between these phenomena. Nonetheless, the dimensions of the gravitational constant, which have not yet been physically interpreted, and the strange Gaussian system of measurements which involves the elimination of mass on account of this relationship between gravity and inertia, give us food for thought.

The relationship of acceleration to an absolute space could be assumed as long as a resting light ether could be used as a reference system. As early as 1883, Mach's mechanics declared that the notion that there could be a privileged reference system independent of material processes is absurd, and gave hints that the conception of an acceleration against space might be an intermediary to one compared to all other masses.[1]

In particular, Mach addressed the argument of absolute mechanics which holds that absolute centripetal accelerations can be identified by the presence of centrifugal forces, and pointed out that these centrifugal forces are only observed in systems of very small extent that are rotating against the fixed stars.

But recently Mr. Abraham and Mr. Mie have argued against Einstein's demand for the covariance of the physical laws with respect to arbitrary transformations of the reference system on the basis that such a covariance would contradict the observed inertial forces.[2]

[1] E. Mach, *Die Mechanik in ihrer Entwickung*, 6th ed., 1908, p. 250—253

[2] Discussion note by G. Mie on Einstein's lecture, this journal. 14, 1264, 1913; Abraham, *Die neue Machanik*, Scienta Jan. 1914, *Sur le probléme de la relativité*, Juli 1914.

Only recently, after I had illustrated the possibility of acceleration-relative mechanics using a concrete case, has Mr. Abraham withdrawn his fundamental objection.

In the essay in question, I stated and for the first time quantitatively formulated the idea that the relativity of acceleration can only be implemented if the centrifugal forces of a rotating body correspond to centripetal forces of all other masses so that there is no dynamical difference between a body that rotates with respect to all other masses and the converse situation in which [180] all other masses rotate with respect to the body.[3]

However, my knowledge of the equality of inertial and gravitational masses had not been included as necessary, since my approach involved separate kinetic and potential energy functions.

Mr. Einstein's equivalence hypothesis which asserts the mechanical and optical identity of an acceleration field with a field of constant gravity seems to imply the deeper meaning that gravity is also a resistance to acceleration. Of course, this idea could not be applied directly to inhomogeneous gravitational fields. The basic assumption of Hertz's principles of mechanics should also be remembered here, according to which all forces should be viewed as inertial forces. However, Hertzian mechanics is completely removed from the idea of the relativity of inertia. It is therefore all the more remarkable that it is precisely this idea that makes it possible to fulfill Hertz's ideal requirement of representing gravity as an inertial force.

In this direction, the appearance of the above-mentioned counterparts of the centrifugal force led me to attempt to make these forces responsible for gravity. If I am successful, grav-

[3] H. Reissner, *Uber die Relativitat der Beschleunigungen in der Mechanik.* This journal. 15, 371 bis 375, 1914.

ity would be understood as a direct and necessary consequence of the relativity of acceleration, the identity of the gravitational and inertial masses would be shown to be self-evident and the gravitational field would not only be equivalent to an accelerated space, as Einstein proposes, but gravity itself would be identified as a resistance to relative acceleration.

First, the following postulates should be derived:

1. The inertial force of mechanics can be represented as the resistance to translational accelerations of a mass relative to all other masses in space.
2. Weight, or Newtonian gravity can be represented as the inertial force of the relative rotation of masses.

The first postulate is in principle already contained in my earlier essay. There the kinetic energy of a closed system of 2 mass points, m_1 and m_2 at a distance r, $T = m_1 m_2 \dot{r}^2 f(r)$ was set. However, at that time I saw no reason not to equate $f(r)$ to a constant and assumed a separate force function for gravitation.

Here, however, the function $f(r)$ of the mutual distance should be used in such a way that no additional force function is required for gravitation.

Progress should therefore initially consist in deriving inertia and gravitation solely from kinetic energy.

The kinetic energy for a system of two masses is given by:

$$T = \frac{1}{2} \sum \mu_s \mu_t \dot{r}_{st}^2 r_{st}^{-1} \tag{1}$$

where r_{st} is the distance between the points s and t with mass constants μ_s and μ_t, and $\dot{r}_{st}$ is the rate of change of this distance. The question of how these distances and speeds are to be measured should be ignored for the time being.

The Lagrangian equations then provide the forces between the mass points:

$$K_{st} = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{r}_{st}}\right) - \frac{\partial}{\partial r_{st}}(T).$$

With

$$\begin{aligned} \frac{\partial T}{\partial \dot{r}_{st}} &= \dot{r}_{st} r_{st}^{-1} \mu_s \mu_t, \\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{r}_{st}}\right) &= \left(\ddot{r}_{st} r_{st}^{-1} - \dot{r}_{st}^2 r_{st}^{-2}\right) \mu_s \mu_t, \\ \frac{\partial T}{\partial r_{st}} &= -\dot{r}_s trst^{-1} - \frac{1}{2} \dot{r}_{st}^2 r_{st}^2 \end{aligned}$$

becomes:

$$K_{st} = \mu_s \mu_t \left(\ddot{r}_{st} r_{st}^{-1} - \frac{1}{2} \dot{r}_{st}^2 r_{st}^{-2}\right) \tag{2}$$

The entire system is now referred to an arbitrary Cartesian coordinate system, so that one can set:

$$\begin{aligned} r_{st} &= \left[(x_s - x_t)^2 + (y_s - y_t)^2 + (z_s - z_t)^2\right]^{1/2} \\ \dot{r}_{st}, &= r_{st}^{-1}\left[(x_s - x_t)(\dot{x}_s - \dot{x}_t) + (y_s - y_t)(\dot{y}_s - \dot{y}_t) + \right. \\ & \quad (z_s - z_t)(z_s - \dot{z}_t), \\ \ddot{r}_{st} &= r_{st}^{-1}\left[(x_s - x_t)(\ddot{x}_s - \ddot{x}_t) + (y_s - y_t)(\ddot{y}_s - \ddot{y}_t)\right. \\ & \quad + (z_s - z_t)(z_s - \ddot{z}_t) - r_{st}^{-1} \dot{r}_{st}^2. \end{aligned}$$

Furthermore, let the force be in the direction of an axis, e.g. the X-axis, determined as the sum of the projections of the radial forces to:

$$\begin{aligned} X_t &= \sum K_{st}(x_s - x_t) r_{st}^{-1} \\ &= \sum \mu_{st}\left(\ddot{r}_{st} r_{st}^{-1} - \frac{1}{2} \dot{r}_{st}^2 r_{st}^{-2}\right)(x_s - x_t) r_{st}^{-1} \end{aligned} \tag{3}$$

If one lets the origin of the coordinate system correspond to the position and velocity of the point t, [181] but not its acceleration, meaning that we choose

$$x_t,\ \dot{x}_t,\ y_t,\ \dot{y}_t,\ z_t, \text{ and } \dot{z}_t = 0$$

then we can write:

$$\ddot{x}_{st} = (-\ddot{x}_t x_s - \ddot{y}_t y_s - \ddot{z}_t z_s) r_{st}^{-1} + \ddot{r}_{st}^0$$

Here $\ddot{r}_{st}^0$ is the counter-acceleration of the point s against the origin of the coordinate system.

This makes the X-component of the force similar to my earlier paper:

$$\begin{aligned} X_t = &-\ddot{x}_t \mu_t \sum \mu_s x_s^2 r_{st}^{-3} - \ddot{y}_t \mu_t \sum \mu_s x_s y_s r_{st}^{-3} \\ &- \ddot{z}_t \mu_t \sum \mu_s x_s z_s r_{st}^{-3} \\ &+ \mu_t \sum \mu_s x_s \left(\ddot{r}_{st}^0 r_{st}^{-1} - \frac{1}{2} \dot{r}_{st}^2 r_{st}^{-3}\right). \end{aligned} \tag{4}$$

Newton's basic equation 'mass times acceleration equals force' applies if we set:

$$\begin{aligned} \mu_t \sum \mu_s x_{st}^2 r_{st}^{-3} = \mu_t \sum \mu_s y_{st}^2 r_{st}^{-3} &= \mu_t \sum \mu_s z_{st}^2 r_{st}^{-3} \\ &= \frac{\mu_t}{3} \sum \mu_s r_{st}^{-1} = m_t \\ \sum \mu_s x_{st} y_{st} r_{st}^{-3} = \sum \mu_s x_{st} z_{st} r_{st}^{-3} &= \sum \mu_s y_{st} z_{st} r_{st}^{-3} = 0 \end{aligned} \tag{5}$$

In addition, one must calculate the last sum of the right-hand side of eq. (4), either by setting it equal to zero and thus using it to determine the movement of the coordinate system, or by using it as an external force, for instance by considering it as a gravitational force or by using this sum partly for one purpose and partly for another.

The equivalence of mechanics for inertial forces and the other forces of nature lies in these various possibilities.

If one were to use the statements of eq. (5), then instead of Newton's scalar theory of inertia with the scalar mass, an initially three-dimensional tensor theory of inertia with the values in eq. (5) specifying 6 components of a symmetrical tensor would arise. It is known that the generalized Einstein-Grossmann theory of relativity, which of course has a much more general starting point, also tends towards this approach.

It should also be noted that the mass

$$m_t = \frac{\mu_t}{3} \sum \mu_s r_{st}^{-1}$$

of a point cannot be a global constant even in a scalar theory, but is rather a function of position. However, for those forces that also turn out to be proportional to the mass, this variability will not be apparent. This variability is common to all acceleration-relative theories.[4]

On the other hand, the fact that classical mechanics does a good job with mass as a constant scalar quantity must probably be taken as an indication that we are in a region of space with a sufficiently symmetrical mass distribution, unless it turns out that in a more general tensorial theory the changing character of the inertial mass appears as unchanging due to the covariance of our measuring instruments. However, since the generalized theory of relativity allows for the possibility of detecting a curvature of our measurement of light rays and a shift of spectral lines in a gravitational field, the second interpretation seems less likely to me. It is also on account of eq. (4) that we may raise the question of whether there might be signs that inertial forces in the plane of the Milky Way are greater than those perpendicular to it.

The fact that one can actually show that the final sum in eq. (4) can be understood as a gravitational effect, should now be proven in detail. The last term of the force expression (4) denotes a force along the line connecting the two mass points of magnitude:

$$K_{st} = \mu_s\mu_t\left(\ddot{r}_{st}r_{st}^{-1} - \frac{1}{2}\dot{r}_{st}^2 r_{st}^{-2}\right) \tag{2}$$

We should now calculate the form that this force takes when two revolving bodies, each rotating about their axis of symmetry, face each other at a distance that is large compared to the dimensions of the bodies.

A mass element $d\mu$ is accelerated away from its axis of rotation by $\frac{v^2}{a}$, where v is the peripheral speed and a is the radius of rotation.

The acceleration component located at the shortest distance between both axes of rotation, initially assumed to be parallel, is then $\frac{v^2}{a}\cos\phi$ (see figure) and the term becomes:

$$\ddot{r}r^{-1} = \frac{v^2\cos\phi}{a(r + a\cos\phi)}$$

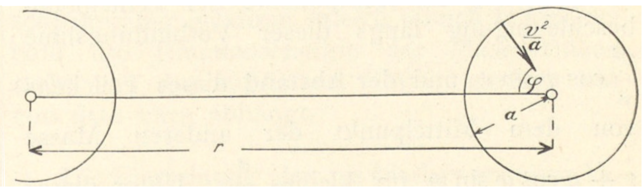

Figure 1: [Translator's note]: Diagram taken from the original paper.

[182]
Integrating over a ring element, we get:

$$\int d\mu\ddot{r}r^{-1} = v^2\nu\int_0^{2\pi}\frac{d\phi\cos\phi}{r + a\cos\phi} \tag{6a}$$

where ν is a line density of μ along the specified sought circle. For small a/r this integral becomes:

$$\frac{v^2\nu}{r^2}\int_0^{2\pi} d\phi\cos\phi(r - a\cos\phi) = \frac{v^2\nu\pi a}{r^2} = \frac{v^2 d\mu}{r^2}$$

Accordingly, $\dot{r} = v\sin\phi$ and

$$\begin{aligned}\int d\mu\dot{r}^2r^{-2} &= v^2\nu a\int_0^{2\pi}\frac{d\phi\sin^2\phi}{(r + a\cos\phi)^2}\\ &\sim \frac{v^2\nu a}{r^4}\int_0^{2\pi} d\phi\sin^2\phi(r - a\cos\phi)^2\\ &= \frac{v^2\nu a\pi}{r^2} = \frac{v^2 d\mu}{r^2}\end{aligned} \tag{6b}$$

The attraction of the whole body with the index s onto another non-rotating body with the mass coefficient μ_t at a distance r becomes:

$$\mu_t\frac{\omega^2}{2r^2}\int a^2 d\mu_s = \frac{1}{2}\mu_t\mu_s\frac{k_s^2\omega_s^2}{r_{st}^2}$$

where k_s is the radius of gyration and ω_s the angular velocity of the body s.

If both masses rotate, the effects add up at small $\frac{a}{r}$ so that the force of attraction becomes:

$$K = \frac{1}{2}\mu_s\mu_t\frac{k_s^2\omega_s^2 + k_t^2\omega_t^2}{r^2} \tag{2a}$$

The form of this force law already corresponds to Newton's gravity, but the attraction of a particle rotating in this way is not uniform in all directions but greatest perpendicular to the axis of rotation and equal to zero in this axis.

If the connecting line (the distance r) of the centers of two masses is at an angle ψ to the axis of rotation of one of the particles, then the component of the centripetal acceleration is along these connecting lines $\frac{v^2}{a}\cos\phi\sin\psi$ and the distance of this particle from the center of the other mass $r + a\cos\phi\sin\psi$ for small $\frac{a}{r}$. According to this assumption, we multiply the integrals (6a), (6b) by $\sin^2\psi$, and if we take the attractive force for the case that the axis of rotation forms the angle ψ with the distance r denoted by K_ψ, we obtain

$$K_\psi = K\sin^2\psi$$

One could now consider the structure of gravitating matter in such a way that rotating particles are distributed in every volume element without any axis of rotation being present.

Then, using the example of 3 rotating particles with mutually perpendicular axes of rotation, it can be seen that the resulting force on another in a prescribed direction must amount to:

$$K_r = \frac{2}{3}\mu_s\mu_t\frac{k_s^2\omega_s^2 + k_t^2\omega_t^2}{2r_{st}^2}. \tag{2b}$$

Here, for example, within μ_s is the sum of all mass coefficients of the individual rotating particles of the mass (of index

[4][In Nordstrom's theory for example the mass is given by: $m = \mu\left(Const - \sum\frac{\mu}{r}\right)$.

s) under consideration. However, the mass under consideration, although it contains perhaps a large number of irregularly distributed rotating particles, has a very small extent compared to the distance to the particles of the second mass of index t, so that r_{st} can be understood as the distance between the centers of gravity of the two masses.

If Newton's law of attraction is to be expressed in (2b), the following relationship must hold:

$$\gamma \frac{m_s m_t}{r^2} = \frac{1}{3} \frac{\mu_s \mu_t}{r^2} (k_s^2 \omega_s^2 + k_t^2 \omega_t^2)$$

where γ is the gravitational constant.

So according to equation (5):

$$\gamma = 3 \frac{k_s^2 \omega_s^2 + k_t^2 \omega_t^2}{\phi_s \phi_t}$$

where ϕ_s, and ϕ_t, are the potential functions $\sum \frac{\mu}{r}$ at the locations of the masses under consideration.

However, the gravitational constant γ could only be a universal constant in a space of such an extent where the variation of the rotational constant contained in the mass unit energy $e = \frac{k^2\omega^2}{2}$ and the potential function ϕ are sufficiently small. Then it would be given by:

$$\gamma = 12 \frac{e}{\phi^2} \tag{7}$$

On the other hand, it was found above, in accordance with Mr. Einstein's theory of gravitation, that the mass value that determines the inertia must also depend on the position in relation to all other masses, or more precisely the [183] potential function $\sum \frac{\mu}{r}$. It is therefore to be expected that the gravitational constant should also depend on this value.

Because of the transition to the theory of relativity, it is now useful to represent the above results using the kinetic energy T of the entire system.

This approach is based on the total energy of all the masses in space:

$$T = \frac{1}{2} \sum \sum \mu_s \mu_t \dot{r}_{st}^2 r_{st}^{-1}.$$

The part relating to a single mass μ_t, excluding any self-interaction, is:

$$T_1 = \frac{1}{2} \mu_t \sum \mu_s \dot{r}_{st}^2 r_{st}^{-1}.$$

Previous considerations have shown that, depending on the nature of the mass distribution, this energy can essentially be broken down into a part that comes only from the movement of the point under consideration and a part that comes solely from the movement of all other points:

$$T_t = \frac{1}{2} \frac{\mu_t}{3} \sum \frac{\mu_s}{r_{st}} q^2 + \frac{1}{2} \mu_t \sum \mu_s \dot{r}_{st}^{0^2} r_{st}^{-1},$$

where $\dot{r}_{st}^0$ are the mean rates of change of the distances from the mass μ_t considered to be at rest with its center of gravity.

The first term gives the classic expression for the kinetic energy of a mass $m_t = \mu_t \sum \frac{\mu_s}{r}$ moving at speed q. Furthermore, according to earlier considerations, the second term must correspond to the gravitational energy if in $\dot{r}$ only the rotations of the masses μ_s, but not their translations, are considered.

The evaluation of this second term has actually already been accomplished by the expletive (2b) found earlier. This requires an energy of the amount

$$\frac{2}{3} \mu_t k^2 \omega^2 \sum \frac{\mu_s}{r},$$

where the energy density of the rotation of all elementary particles relative to the unit mass must be assumed to be the same. The complete energy expression is then:

$$T_t = \frac{1}{2} \frac{\mu_t}{3} \sum \frac{\mu_s}{r} \left(q_t^2 + 4k^2\omega^2 \right) \tag{1a}$$

From this expression we get both the inertial force and the force of gravity in the form:

$$\text{Inertial force:} \quad K_t = \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}} \right) = \frac{d}{dt} (m\dot{x}), \tag{4a}$$

$$\text{Gravitational force:} \quad K_g = \frac{\partial T}{\partial x} = \frac{2}{3} k^2 \omega^2 \mu_t \frac{\partial}{\partial x} \sum \frac{\mu_s}{r}. \tag{4b}$$

This expletive would perfectly match the Newtonian if the same proportionality between mass coefficient μ and mass m existed everywhere. However, this is only approximately the case if the potential function $\sum \frac{\mu}{r}$ changes gradually enough. Using that

$$\mu_t = m_t \frac{3}{\phi},$$

where

$$\phi = \sum \frac{\mu_s}{r},$$

we get:

$$K_g = 6\, k^2 \omega^2 \frac{m_t}{\phi} \frac{\partial}{\partial x} \left(\sum \frac{m_s}{\phi r} \right)$$
$$= m_t 6 \frac{k^2\omega^2}{\phi} \left[\frac{1}{\phi} \frac{\partial}{\partial x} \sum \frac{m_s}{r} - \frac{1}{\phi^2} \sum \frac{m_s}{r} \frac{\partial \phi}{\partial x} \right].$$

If $\phi = \sum \frac{\mu}{r}$ is now assumed to be large and $\frac{\partial \phi}{\partial x}$ to be small, a first-order approximation results in

$$K_g = 6 \frac{k^2 \omega^2}{\left(\sum \frac{\mu}{r} \right)^2} m_t \frac{\partial \sum \frac{m}{r}}{\partial x},$$

so the gravitational constant as above is:

$$\gamma = 6 \frac{k^2 \omega^2}{\left(\sum \frac{\mu}{r} \right)^2}. \tag{7}$$

Here too, as in all newer theories of gravity, the Laplace-Poisson potential equation of gravity is only approximately valid, although this approximation must turn out to be extremely accurate.

Furthermore, the forms of force (4a) and (4b) fulfill the condition that at all points of the gravitational field all masses experience the same acceleration (fall at the same speed), as soon as the rotational energy of the mass unit is not related to its material composition, but only depends on the location.

It is desirable to classify the results achieved so far into a field theory that also includes the changes over time and satisfies the postulate of relativity. [184]

Now, according to the note on page 181, it is certain that a connection cannot be made to Nordström's scalar theory of gravity, since there the inertial mass decreases as other masses approach, whereas in our theory it increases similarly to Einstein's theory. It also seems that the character of the above approach points more towards a tensor theory.

However, I have not yet been able to fully connect with Einstein-Großmann's generalized relativity scheme. It seems to me that this is difficult for the following reason.

The complete differential equations of the gravitational field and the complete covariant stress-energy tensor of the mass-flow in Einstein's last publications, which together form the generalization of the Laplace-Poisson potential equation, represent a mathematically very difficult problem. However, Einstein himself gains from them nevertheless, he still obtained valuable results by using the line element of the previous theory of relativity as a first approximation and finding a correction assumed to be small from the energy tensor of this first approximation using the now linear differential equations of the field.

Through this procedure he consciously foregoes any insight into the mechanical structure of the initial values of the line element, which he takes as given, even though they would have to follow from the differential equations. However, it is precisely this physical idea that is provided by the approach given here, even if only for the equilibrium of the field, which can perhaps only be integrated into Einstein's general field equations after a different integration method. I believe in such a connection because my results regarding the dependency between inertia, the potential function and the speed of light are built in a very similar way to Einstein's, and Einstein's scheme must be of wide applicability.

In what follows, I will provide a scalar approach to a field theory, which is obeyed in sufficiently small regions of the former recent theory and contains the above results as the first approximation. I set the line element

$$ds = dt\left[c_0^2 - \frac{\phi}{\phi_0}\left(4k^2\omega^2 + q^2\right)\right]^{1/2},$$

where c_0 is a very large constant, q is the speed of the point under consideration, and ϕ is the four-dimensional potential obeying the equation

$$\Box\,\phi = -4\pi Q,$$

and ϕ_0 is the value of ϕ at the coordinate starting point. Let Q be the frame mass density of μ and $k^2\omega^2$ should again be treated as a constant.

The Lagrangian function has the value:

$$H = -\mu\frac{\phi_0}{3}c_0\frac{ds}{dt} = m_0 c_0\frac{ds}{dt}.$$

The inertial force then becomes:

$$K_t = \frac{d}{dt}\left(\frac{\partial H}{\partial \dot{x}}\right) = \mu\frac{d}{dt}\left[m\dot{x}\left(1 - \frac{\phi}{\phi_0}\{4k^2\omega^2 + q^2\}\right)\right]^{1/2},$$

The gravitational force takes the value:

$$K_g = \frac{\partial H}{\partial x} = \frac{\mu}{3}\frac{\partial \phi}{\partial x}\left(2k^2\omega^2 + \frac{q^2}{2}\right)\left(1 - \frac{\phi}{\phi_0}\{4k^2\omega^2 + q^2\}\right)^{1/2},$$

The previous forces of eq. (4a) and (4b) obviously represent the first approximation of these latter force expressions, which mean an extension of Newton's force law for the case of finite velocity.

If one further assumes, as in the theory of relativity, that the line element $ds = 0$ results in a speed q equal to the speed of light, then this becomes

$$c = \sqrt{c_0^2\frac{\phi_0}{\phi} - 4k^2\omega^2}$$

Which therefore decreases as one approaches a mass. c_0 is the speed of light at the starting point if there is no mass rotation.

The energy of a mass point is also obtained as a generalization of the earlier energy expression (1a) using the usual ansatz

$$E = \frac{\partial H}{\partial \dot{x}}\dot{x} + \frac{\partial H}{\partial \dot{y}}\dot{y} + \frac{\partial H}{\partial \dot{z}}\dot{z} - H = mc^2\left(1 - \frac{\phi}{\phi_0}\left(4k^2\omega^2 + q^2\right)\right)^{1/2}$$

The energy value also decreases as it approaches other masses.

The above expressions derived from the four-dimensional line element becomes the well-known expressions of the Einstein-Minkowski theory of relativity with the starting point of the coordinate system in the absence of mass rotation in the known distribution of all other masses. [185]

The gravitational forces that arise when masses rotate under the assumption of the relativity of inertia are due to the fact that in every rotation the centripetal accelerations that generate attraction are closer to all other masses than those that produce repulsion. The often asked question about the possibility of negative masses is therefore completely negated.

This mode of operation of rotation is also inherent in other forms of movement if one allows their temporal average values instead of the forces and accelerations. Every collection of mass particles that somehow move through one another must exert a gravitational attraction on other masses, as long as the relativity of inertia assumed above is correct. The calculation would depend on the temporal and spatial averages of $\mu\dot{r}^2 r^{-1}$ and the average effect would be equivalent to that of a rotation of a certain amount.

Finally, one can now raise the question of whether the general rotation of all mass particles can also be interpreted from the perspective of the relativity of inertia. In fact, every torque of the inertial forces must be absorbed by corresponding torques of all other masses, so that every change in the angular motion of one body must be followed by a corresponding change in the angular motion of all other bodies. There must therefore be a certain balancing of the rotations of all masses.

Summary

An earlier approach to the acceleration-relative form of the kinetic energy of masses is simplified in such a way that the gravitational effect appears as a pure inertial effect without the addition of potential energy.

However, this requires the hypothesis that all masses that excite a gravitational field have rotation and that the axes of rotation are irregularly directed and distributed. The gravitational force is then represented as a mutual centripetal force.

With this approach, the inert mass is only found approximately as a scalar, if one assumes sufficient symmetry in the mass distribution of space. The size of both inertial and attractive masses depends in a very specific way on the distribution of matter.

While the first approach of an elementary law of two masses seems to correspond to a tensor theory of inertia and gravity. A scalar theory of gravity is established, assuming a certain symmetry of the mass distribution and mass rotation of our space, and the essential results, previously understood as long-distance effects, are found again with the help of a Lagrangian function of the field. This results in an increase in the inertial mass and a reduction in the speed of light and energy when masses approach.

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